THEORETICAL NAVIGATION

AND

NAUTICAL ASTRONOMY.

CLARK.

VAN NOSTRAND

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INTRODUCTION.

The following pages have been prepared for use at the U. S. Naval Academy.

Napier's and Bowditch's Rules have been used in deducing the formulæ, which are generally those used in Bowd. Nav.

References to Trigonometry are to the treatise of Prof. Chauvenet.

Not seeing any good reason for making distinctive "Sailings" while still considering the earth's surface as a plane, the author has taken the liberty of placing them together under the head of "Common Sailing."

For the method of deducing the equation of "Mercator's Sailing" the thanks of the author are due to Prof. J. M. Rice, of the Naval Academy.

CHAPTER I.

DEFINITIONS AND NOTATION.

1. Meridians are great circles of the sphere, passing through both poles.

2. Suppose a ship to sail so that the line of her keel makes a *constant* angle with each successive meridian; this line is called the *ship's track* or loxodromic curve. In old nautical works, the *rhumb line.*

3. The constant angle made by this line with each meridian is called the *true course.* In the following problems the word course will be understood to mean true course, and will be denoted by C.

4. The compass needle, undisturbed by local causes, points to the magnetic pole, and great circles passing through this pole are called *magnetic* meridians. The angle which the loxodromic curve makes with the magnetic meridians is called the *magnetic* or compass course. Compass course must be reduced to true course previous to the solution of nautical problems in which course is considered.

5. The portion of the loxodromic curve considered in any problem, is called the *distance.* It is necessarily the number of miles passed over by the vessel on the course which belongs with it.

6. Latitude is angular distance north or south of the equator, measured in degrees, minutes, etc., of a great circle, denoted by L.

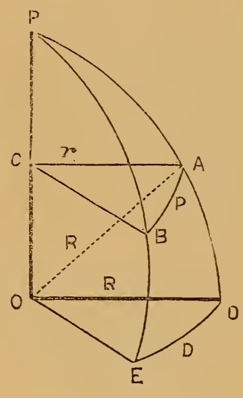
7. Difference of latitude, denoted by l, is the portion of a meridian included between two parallels of latitude.

8. **Longitude** is the angular distance between any meridian and a fixed or prime meridian. The prime meridian is usually that of Greenwich. It may be considered as angle at the pole, of which the corresponding portion of the equator is a measure. It is denoted by λ.

9. **Difference of longitude** is angle at the pole, or the corresponding arc of the equator between any two meridians, represented by D.

10. **Departure** is the angular distance between any two meridians measured on any parallel of latitude. As parallels of latitude vary in size, the units (degrees, etc.) become smaller. If, however, we have departure determined in angular units of its own circle, the corresponding difference of longitude would be the same. Departure is, however, found in the linear value of units of a great circle of the sphere. In order, then, to determine the corresponding difference of longitude, it will be necessary to know first the relation between the units of any parallel of latitude and the corresponding units of the equator.

FIG. 1.



11. To find these relations, we have in Fig. 1

$$E\,D = D$$
$$A\,B = p \text{ the departure in Lat. L.}$$

D and p are similar arcs of circles, and therefore are to each other as their radii.

$$p = \frac{Dr}{R}$$

$$C\,A\,O = A\,O\,D = L \quad \frac{r}{R} = \cos L$$

which substituted in above gives

$$p = D \cos L \text{ or}$$
$$D = p \sec L.$$

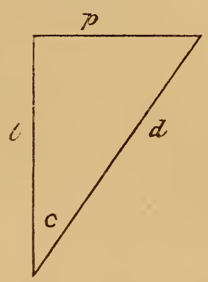
which give the required relations.

Having therefore the departure expressed in units of the equator (in nautical miles), we find the corresponding difference of longitude by multiplying it by the secant of the latitude in which the departure is situated.

COMMON SAILING.

12. For such small distances as an ordinary day's run at sea, it is customary to consider the small portion of the earth's surface passed over as a plane. The difference of latitude and departure corresponding to the course and distance sailed are determined by the solution of a plane right angled triangle.

FIG. 2.



In Fig. 2

the difference of latitude $l = d \cos C$
the departure $p = d \sin C$
$p = l \tan C$

This is sufficiently accurate for small distances.

These equations are employed in what is called by navigators "working dead reckoning." Their computation is facilitated by the use of Tables I. and II. Bowd., which are tables for the solution of any plane right triangle, calling the distance *hypothenuse*, difference of latitude *side adjacent*, and departure *side opposite*. When several courses are sailed, the triangle is solved separately for each value of C, and the algebraical sum of l', l'', l''', etc., p', p'', p''', etc., are taken for the whole difference of latitude and the whole departure.

13. The equations above are strictly true when

$$\mathrm{d}\, l = \mathrm{d}\, d \cos C.$$
$$\mathrm{d}\, p = \mathrm{d}\, d \sin C.$$
$$\mathrm{d}\, p = \mathrm{d}\, l \tan C.$$

The smaller l, d, and p are taken, therefore, the nearer correct will be the result.

The departure p, formed from the sum of several partial departures, is, of course, for different latitudes. It is customary to assume it upon the middle parallel. That is, the middle latitude is found (in the figure) between each extremity of l and the departure assumed upon it. The difference of longitude is found from

$$D = p \sec L.$$

L being this middle latitude,

$$L = L' + \tfrac{1}{2}\, l.$$
$$L = L'' - \tfrac{1}{2}\, l.$$

The difference of latitude found being applied to the *latitude left*, with proper sign gives *latitude in.*

The difference of longitude applied to *longitude left*, with proper sign will give *longitude in.*

14. Several problems arise in Common Sailing which are solved on the supposition that the triangle is a plane right triangle. They are solved generally *by inspection* of Tables I. and II. They may be solved by logarithms, using some form of the preceding equations.

The two following are selected as examples :

15. Problem 1. To find current.

The difference between the latitudes as found by observation and by "dead reckoning," is taken, and also the difference between the longitudes as determined in same manner. The observed position is considered as the correct position, and any difference in the two positions may be due to current.

The difference of longitude is changed to departure by

$$p = D \cos L.$$

The course or direction of the set of the current is then determined by

$$\tan C = \frac{p}{l}$$

and its *amount* or distance by

$$d = \frac{p}{\sin C}$$

or

$$d = \frac{l}{\cos C}$$

16. Problem 2. To find the course and distance "made good."

The difference between the latitude left and that arrived at (by observation) is taken.

The difference between the longitudes is changed, as in preceding problem, to departure. The same equations are then solved as before, C being in this case the course made good, and d the distance made good.

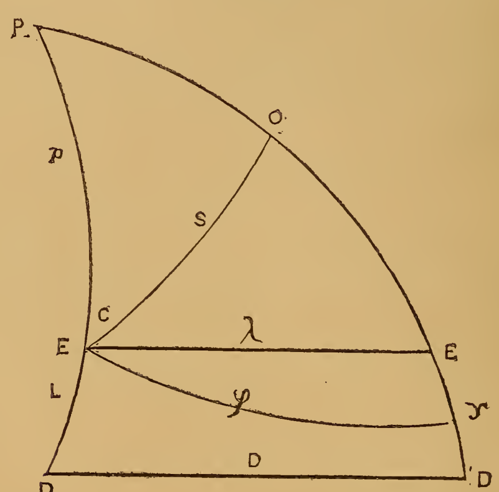
This problem, as we shall see, is more correctly solved by Mercator's Sailing.

CHAPTER II.

MERCATOR'S SAILING.

1. We have, in Common Sailing, considered a small portion of the earth's surface as a plane. This is sufficiently correct for small distances as an ordinary day's run. A more rigorous solution of problems appertaining to the loxodromic curve is necessary.

FIG. 3.



In Fig. 3, $E\,O$ is a portion of the loxodromic curve. $E\,E'$ a parallel of latitude passing through origin. $E\,y$ a great circle of the sphere through the same point. p equals $P\,E$ the co-Lat. of E. $P\,E\,O = C$, the course.

Decompose s along p and ϕ, and we have

$$\text{Cot } C = -\frac{d\,p}{d\,\phi}$$

λ and ϕ have a common tangent at the point E, and as $d\,\lambda$ and $d\,\phi$ denote angular velocities, they are to each other as the corresponding radii.

$$\therefore\ d\,\phi = d\,\lambda \cos L, \text{ or } d\,\phi = d\,.\,\lambda \sin p.$$

and
$$\cot C = -\frac{d\,p}{d\,\phi} = -\frac{d\,p}{\sin p\,d\,\lambda}$$

$$\cot C \int_{\lambda_2}^{\lambda_1} d\,.\,\lambda = -\int_{p_2}^{p_1} \frac{d\,.\,p}{\sin p} = -\int_{p_2}^{p_1} \frac{\frac{1}{2}\,d\,.\,p}{\sin \frac{1}{2} p \cos \frac{1}{2} p} \qquad (a.)$$

Dividing numerator and denominator by $\cos \frac{1}{2} p$ and integrating first member.

$$\operatorname{Cot} C\,(\lambda_1 - \lambda_2) = -\int_{p_2}^{p_1} \frac{\sec^2 \frac{1}{2} p\,d\,.\,\frac{1}{2} p}{\tan \frac{1}{2} p} = -\log \tan \tfrac{1}{2} p \Big]_{p_2}^{p_1} \qquad (b.)$$

But

$-\log \tan \frac{1}{2} p = \log \cot \frac{1}{2} p = \log \cot \frac{1}{2}\,(90° - L) = \log \tan\,(45° + \frac{1}{2} L)$ which substituted in (b) gives

$$\operatorname{Cot} C\,(\lambda_1 - \lambda_2) = \log \tan\,(45° + \tfrac{1}{2} L)\Big]_{L_2}^{L_1} \qquad (c.)$$

the limits L_1 and L_2 changing for those of p_1 and p_2.

$$\lambda_1 - \lambda_2 = \tan C \log \frac{\tan\,(45° + \frac{1}{2} L_1)}{\tan\,(45° + \frac{1}{2} L_2)}$$

If $L_2 = 0$, or the point E be at the equator

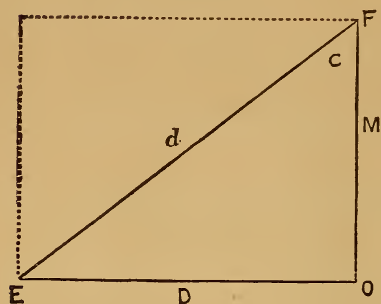
$$\lambda_1 - \lambda_2 = D = \tan C \log \tan\,(45° + \tfrac{1}{2} L).$$

In this the logarithm is Naperian, and D is in angular measure. To change to the common system of logarithms, we multiply by the reciprocal of the *modulus*, $\frac{1}{m} = 2.302585093$; and to change D for the globe, multiply by the radius of the earth in minutes $= 3437.74677$, and we have

$$D = 7915.70447 \log \tan\,(45° + \tfrac{1}{2} L) \tan C.$$

2. The relation existing between D and C in this expression is that of an angle and side opposite in a plane right triangle and may be represented as in Fig. 4 in which

Fig. 4.



D is the difference of longitude. C is the course, and the side $F\,O = 7915.70447$ log tan. $(45° + \frac{1}{2}L)$. $F\,O$ is called the *Augmented Latitude,* and will be represented by M.

In the expression

$$M = 7915.70447 \log \tan \left(45° + \tfrac{1}{2} L\right)$$

different values of L may be assumed, and the corresponding augmented latitude computed and tabulated. Table III. Bowd. is such a table, computed for each minute of L from 0° to 84°.

3. From the foregoing, we see that any portion of the loxodromic curve, or ship's track, may be represented by a straight line, as $E\,F$, in Fig. 4. Charts constructed on these principles are called Mercator's Charts. By means of a Table of Augmented Latitudes they are easy to construct, and possess the advantage of showing the ship's track by a straight line, and the course being represented by the angle which this line makes with any meridian.

4. **Problem 1.—A ship sails from a latitude L′ until she arrives at a latitude L″, upon a given course C, find the difference of longitude D.**

For the difference of longitude from where the loxodromic curve intersects the equator, to its intersection with the meridian of L', we have

$$D' = M' \tan C,$$

and to the second latitude

$$D'' = M'' \tan C,$$

$$D = D'' - D' = (M'' - M') \tan C.$$

Find, from a table of augmented latitudes, or by computation, M″ and M′ corresponding to L'' and L' respectively, and take their difference. This is called the *augmented difference* of latitude. Representing it by m, we have

$$D = m \tan C.$$

5. **Problem 2.—Given the latitudes and longitudes of two places; find the course, distance, and departure.**—(Bowd., p. 79, Case I.)

L' and L'' being given, find M' and M'' by computation, or by Table III., Bowd.

$$l = L'' - L', m = M'' - M' \qquad D = \lambda'' - \lambda',$$

by Mercator's Sailing

$$\text{Tan } C = \frac{D}{m}$$

and from Common Sailing

$$d = l \sec C$$
$$p = l \tan C$$

6. In Common Sailing we find the difference of longitude by taking the departure upon the middle parallel of latitude. The proper parallel is one situated nearer the pole. Strictly the departure should be taken upon

$$L\,m = \tfrac{1}{2}\,(L' + L'') + \Delta\, L,$$

To find $\Delta\, L$ (see Tables, Bowd., p. 76, and Stanley, p. 338.)

In Common Sailing we have,

$$\text{Cos } L_m = \frac{p}{D} \qquad (a.)$$

From Common Sailing

$$p = l \tan C,$$

and from Mercator's

$$D = m \tan C$$

which substituted in (a) gives

$$\text{Cos } L_m = \frac{l}{m} \qquad (b.)$$

$$\therefore 1 - 2 \sin^2 \tfrac{1}{2} L_m = \frac{l}{m}$$

$$\text{Sin } \tfrac{1}{2}\, L_m = \sqrt{\frac{m-l}{m}} \qquad (c.)$$

For different values of $L' + L''$ we may find $l = L'' - L'$. The middle latitude used in Common Sailing, is

$$L_o = \tfrac{1}{2}(L' + L'')$$

L_m being found by (c) we have

$$\Delta L = L_m - L_o.$$

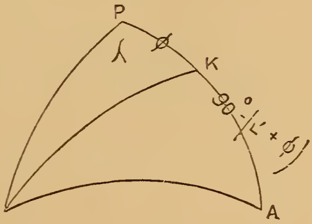
CHAPTER III.

GREAT CIRCLE SAILING.

1. The shortest distance between any two points on the globe, is the arc of a great circle joining them. In running long distances, it is best to follow the arc of a great circle. Strictly speaking, this would be impossible, as the course would have to be changed each instant. It is customary to determine certain points of the great circle, and run from point to point on a loxodromic curve. Circumstances of wind and weather must govern the navigator in choosing his route. Most of the convenient great circle routes have already been computed. A knowledge of Great Circle Sailing is necessary, however, in order to know which tack to put the ship upon in case of adverse winds.

2. **Problem 1.—The latitudes and longitudes of two places being given, to find distance and course from one or both of them.**

FIG. 5.



In Fig. 5, we have given

$$P\,A = 90° - L'$$
$$P\,B = 90° - L''$$
$$A\,P\,B = \lambda = \lambda'' - \lambda'$$
$$P\,A\,B = C, \text{ the course from } A$$
$$P\,B\,A = C', \text{ the course from } B$$

Letting fall the perpendicular $B\,K$ and representing $P\,K$ by ϕ, we have

$$A\,K = 90° - (L' + \phi)$$

By Napier's Rules

$$\text{Cos } \lambda = \tan \phi \tan L''$$
$$\text{Tan } \phi = \cos \lambda \cot L''$$

By Bowd. Rules, or by Napier's Rules and eliminating the perpendicular,

$$\text{Cot } C = \cos (L' + \phi) \cot \lambda \operatorname{cosec} \phi \qquad (b.)$$

and in triangle $B\ A\ K$,

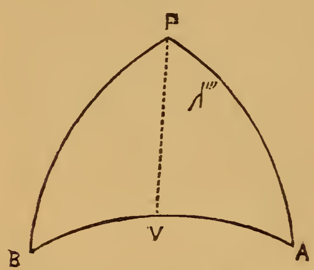
$$\text{Cot } d = \cos C \tan (L' + \phi) \qquad (c.)$$

(a), (b) and (c) furnish the required solution, giving, however, the course from A. To find course from B, change the notation.

3. Problem 2.—To find the highest latitude of the great circle, and longitude of this point from either place.

In Fig. 6, let fall the perpendicular $P\ V$.

Fig. 6.



This perpendicular intersects the great circle at its highest latitude.

We will have by Nap. Rules :

$$\begin{aligned} \text{Sin } L' &= \cot \lambda''' \cot C \\ \text{Cot } \lambda''' &= \tan C \sin L \qquad (a.) \end{aligned}$$

$$\begin{aligned} \text{Cos } \lambda''' &= \cot L \tan L' \\ \text{Cot } L &= \cos \lambda''' \cot L' \qquad (b.) \end{aligned}$$

(a) and (b) solve the problem, giving λ''' from A. To find the longitude we have

$$\lambda_0 = \lambda''' + \lambda'.$$

4. Problem 3.—To find latitudes and longitudes of intermediate points of the great circle.

In Fig. 6, assume longitudes at pleasure on each side of the vertex.

Suppose we assume them 5°, 10°, 15°, etc., upon each side of vertex.

We will have

$$\begin{aligned} \text{Cos } 5° &= \cot L \tan L_1 \\ \text{Cos } 10° &= \cot L \tan L_2 \\ &\text{etc.} \end{aligned}$$

$$\left.\begin{aligned} \text{Tan } L_1 &= \tan L \cos 15° \\ \text{Tan } L_2 &= \tan L \cos 10° \end{aligned}\right\} a$$

$$\text{etc.}$$

Equations (a) give the required solution.

5. In Problem 1, if the perpendicular fall without the triangle, ϕ will be greater than $(90° - L')$ and $KA = \phi - (90° - L')$.

The course is determined in degrees and minutes, and is measured from the meridian of L'. Attention must be paid to the signs of ϕ and λ. The distance is also found in degrees and minutes of the great circle. Reduce to minutes for distance in nautical miles.

6. Having found the latitudes and longitudes of as many points of the great circle as are desired, plot them on chart, and by hand trace through these points the curve; owing to the principles of construction of a Mercator's chart, this will be an irregular curve except when coincident with the equator or a meridian.

CHAPTER IV.

TIME.

1. Time is the hour angle of some heavenly body whose apparent diurnal motion is taken as a measure.

The instant when any point of the celestial sphere is on the meridian of the observer is called *transit.*

2. **Sidereal time** is the hour angle of the first point of Aries (γ). The instant of its transit is sidereal noon, 0 h.

Right ascension is the angular distance of a heavenly body from the first point of Aries reckoned towards the east. Hence when any heavenly body is on the meridian of a place its *R. A.*= the sidereal time.

As the earth revolves 360° in order to bring any meridian two successive times under (γ), we can find the space passed over in one hour by dividing 360 by 24, equals 15°. Hence when the *H. A.* of γ is 15° the sidereal time is 1 h. The interval between two successive transits of γ is the sidereal day. Evidently the interval between two successive transits of any fixed point over the same meridian would be equal in length to a sidereal day.

3. **Apparent time** is the hour angle of the true sun.

The true sun has motion in R. A., and therefore is not a fixed point in the celestial sphere. Its motion is not uniform in the ecliptic, and this of itself would tend to make apparent solar days irregular in length. Besides, as the plane of the ecliptic is inclined to the plane of the equator, the true sun's apparent daily path is not perpendicular to the plane of the meridian; in other words, the true sun approaches the meridian at a constantly varying angle; this also tends to cause irregularity of apparent time. Instruments cannot be constructed to keep apparent time, and astronomers have resorted to the following device in order to obtain a uniform time.

4. **Mean time** is the hour angle of a mean sun (supposed) which has for its annual path, the celestial equator. A first mean sun is supposed to move in the ecliptic at a uniform rate, so as to return to *perigee* and *apogee* with the true sun. This obviates the first difficulty mentioned. The changes in longitude of this mean sun are equal in equal times, but equal changes in longitude do not give equal changes in R. A. So a *second mean sun* (sometimes called simply the mean sun) is supposed to move in the equator at the same rate that the first moves in the ecliptic, and to return to the vernal equinox with it. The time therefore denoted by this second mean sun, although not equal to sidereal time, is perfectly uniform in its increase. The daily difference will evidently be equal to the daily increase in the right ascension of this mean sun = 3 m. 56 s. The instant of transit of the true sun over the meridian of the observer is called *apparent noon.* The instant of transit of mean sun is *mean noon.*

5. **The equation of time** is the difference between apparent and mean time. It is also the difference of the hour angles of true and mean suns. It is also the difference between the right ascensions of the true and mean suns. From what has preceded, we know that the first mean sun's longitude, or, as it is sometimes called the *true sun's mean longitude,* is equal always to the right ascension of the mean sun. Hence the equation of time *is equal to the difference between the true sun's right ascension and its mean longitude.*

6. **Astronomical time** commences at noon or 0 hrs., and is reckoned to the westward 24 hrs. An astronomical day (apparent or mean) is the interval between two successive transits of the sun (apparent or mean).

7. **Civil time** commences at midnight 12 hrs. before the commencement of astronomical time, and is divided into two periods of 12 hrs. each, marked A. M. and P. M.

7. To convert civil into astronomical time.

Remember that the civil day of same date commences 12 hrs. before the astronomical day.

9. Time at different meridians.

It is evident that as any time at one meridian is the H. A. of the heavenly body or point whose motion is considered, to find the corresponding time at any other meridian it is only necessary to add or subtract the angle between the two meridians. In Nautical Astronomy it is generally necessary to convert the given local time to the corresponding Greenwich time, in order to inpolate quantities from the Nautical Almanac, which are computed for the meridian of Greenwich.

10. Having given the local time of any meridian, to find the corresponding Greenwich time.

To the local time add the longitude if west, and subtract if east; the result will be the corresponding Greenwich time of the same kind as the given local time. Conversely, the difference between the time at two meridians (of the same kind) will be the difference of longitude expressed in time. Remembering that 1 h. = 15°, this may readily be converted to arc.

11. To convert apparent time, at a given meridian, into the mean time, or mean into apparent.

If M = the mean time
A = the corrresponding time
E = the equation of time

we have from Art. 5

$$M - A = E$$
$$M = A + E$$
$$A = M - E$$

E may be $\pm$ according as the apparent is greater or less than the mean time. E is found on Page II. of the American Nautical Almanac for Greenwich Mean Noon, and is to be interpolated to the instant of the given Greenwich mean time. Where the given Greenwich time is *apparent time*, then E must be taken from Page I.

12. To change sidereal into solar time it will be first necessary to know the relative value of their units.

In consequence of the earth's annual revolution about the sun, there will be *one less* transit of the sun across any meridian than there will be of any fixed point outside of the earth's orbit, during the period of this revolution.

There are in one year

3662.4222 sidereal days.
365.24222 solar days.

whence we have

$$1 \text{ sid. day} = \frac{365.24222}{366.24222} \text{ sol. day} = 0.99726957 \text{ sol. day},$$

or 24 hrs. sid. time =23 hrs. 56 m. 4.091 s. solar time. And

$$1 \text{ sol. day} = \frac{366.24222}{365.24222} \text{ sid. day} = 1.00273791 \text{ sid. day}.$$

or 24 h. solar time = 24 h. 3 m. 56.555 s. sid. time.

From these relative values Tables II. and III. of the American Nautical Almanac are computed. The first is for converting an interval of sidereal time to the corresponding interval of mean time. The second for changing an interval of mean time into the corresponding interval of sidereal time.

It is evident that, in Table II., the corrections are nothing more than the changes in right ascension of the mean sun during the given intervals of *sidereal time.* It is this change in right ascension which causes the different values of the units. In Table III., the corrections are the changes in right ascension of the mean sun in the given intervals of *mean time.*

13. To convert an interval of sidereal time into the corresponding interval of mean time.

Enter Table II. with the sidereal interval, as an argument. Find the *change* in R. A. of the mean sun and *subtract* this change from the given sidereal interval.

14. To convert an interval of mean time into the corresponding interval of sidereal time.

Enter Table III. with the mean time interval, as an argument.

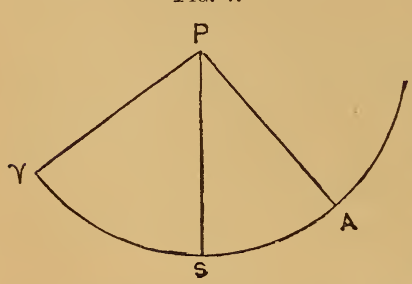
Find the change of R. A. of the mean sun, and *add* this change to the given mean time interval.

15. We have now found a means of changing an *interval* of one kind into an interval of another. It is frequently necessary to find the corresponding *time*, having given another time. To do this it will be necessary to be able to find the R. A. of the mean sun at any instant. The R. A. of the mean sun is given in N. A. for the instant of Greenwich mean noon (Page II. of the month), marked "sid. time, or R. A. of mean sun." This being given for the instant of Greenwich, mean noon must be interpolated to the instant of Greenwich mean time. Hence we have

16. Given the local mean time at any meridian, to find the corresponding sidereal time.

Convert the local mean time into Greenwich mean time, by applying the longitude in time. Enter Table III. of the N. A., and find the change in R. A. of the mean sun for the elapsed Greenwich time; add this to the R. A. given on Page II. of the month for the *preceding* Greenwich noon, and result will be the correct R. A. of mean sun at the instant of time given.

Fig. 7.



Then, in Fig. 7,

$\gamma P S = R A$ mean sun

$A P S = H A$ mean sun or LMT

and $A P \gamma = H A$ of γ or $L S T$

Hence $A P \gamma = A P S + \gamma P S$, or

The sidereal time is equal to the mean time plus the R. A. of mean sun.

17. Given the local sidereal time, at any meridian, to find the corresponding mean time.

Convert the local sidereal time into Greenwich sidereal time, by applying the longitude.

Enter Table II. of the N. A., and find the change in R. A. of the mean sun for the elapsed Greenwich sidereal time. From Page III. of the month, take the "mean time of *preceding* sidereal noon" (which is evidently 24 hrs. minus R. A. of mean sun at that instant). Subtract from this the correction obtained from Table II., and the result is the correct *negative* R. A. of the mean sun at the given sidereal instant.

Then, in Fig. (7.),

$$A\ P\ \gamma = \text{given } L.\ S.\ T.$$
$$\gamma\ P\ S = R.\ A. \text{ of mean sun, and}$$
$$A\ P\ S = \text{the } L.\ M.\ T.$$

$$\text{Hence } A\ P\ S = A\ P\ \gamma - \gamma\ P\ S.$$

We have obtained $\gamma\ P\ S$, however, negatively, and $A\ P\ S = A\ P\ \gamma +$ the corrected negative R. A. of mean sun.

The mean time is equal to the sidereal time, minus the R. A. of mean sun, or plus the negative R. A.

18. Given the apparent time at any meridian, to find the corresponding sidereal time.

Change apparent to mean time (Art. 11.), and proceed as in Art. 16, or

Apply longitude to local apparent time, giving Greenwich apparent time.

Find R. A. of true sun on page I., N. A., and correct by means of given hourly difference to the instant of Greenwich apparent time.

Then in the Fig. (7.)

$$\gamma\ P\ S = \text{R. A. of true sun}$$
$$A\ P\ S = \text{given } L.\ A.\ T. \text{ and}$$
$$A\ P\ \gamma = A\ P\ S + \gamma\ P\ S, \text{ or}$$

The sidereal time is equal to apparent time plus the R. A. of the true sun.

19. Given the sidereal time at any meridian, to find the corresponding apparent time.

Proceed as in Art. 17, then change the mean time to apparent by Art. 11.

20. Given the hour angle of a star, at any meridian, to find the local mean time.

Find in the N. A. the R. A. of the star. To this apply the H.

A. of star plus, when west of the meridian, and minus when east. The result is local sidereal time.

Then proceed as in Art. 17.

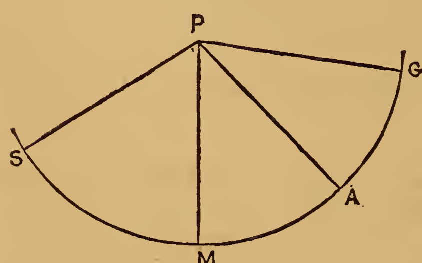
21. To find the hour angle of a star at a given meridian and mean time.

Find the corresponding sidereal time by Art. 16. To this apply the star's R. A.; the difference is star's H. A. + when the sidereal time is greater than the R. A., — when R. A. is greater than sidereal time.

22. Given the hour angle of the moon at any meridian to find the local mean time.

Apply the H. A. of moon to the longitude of the place, which gives the longitude of place which has the moon on its meridian. The N. A., page IV., of the month gives the time of moon's meridian passage at Greenwich, or the angle between the moon and sun. The hourly difference multiplied by difference in time (longitude), and result added to the Greenwich time of passage when longitude is west, subtracted when east, gives the local time of meridian passage, or the corrected angle between the sun and moon. We now have the time at the place which has the moon on its meridian. Applying H. A. of moon gives the time at the given meridian.

FIG. 8.



In Fig. 8

$APM = HA$ of moon

$APG + APM =$ Long. of meridian P. M. from Greenwich.

Having found MPS as stated, $APS = APM + MPS$.

If the Greenwich time be given and longitude APG required. Find R. A. of moon from N. A. and correct for Greenwich time, and proceed as in case of star. Art. 20.

23. To find the hour angle of the moon at any meridian and time.

Proceed as in case of star. Art. 21.

24. Given the hour angle of a planet at any meridian, to find the local mean time.

The N. A. gives the time of meridian passage of each of the planets over the Greenwich meridian, and the local mean time may be found as in first of Art. 22.

If Greenwich time be given and not the longitude, proceed as in second part of Art. 22.

25. To find the hour angle of the sun at a given meridian and time.

The hour angle of the sun is the L. A. T. Proceed as in Art. 11, for changing mean to apparent time. If the apparent time be more than 12 hrs., subtracting it from 24 gives the negative H. A.

26. To find the time of meridian passage of any celestial body, the longitude of the place or Greenwich time being given.

It is only necessary to find from the N. A. the R. A. of the body for the Greenwich time. This R. A. is the sidereal time of transit, change this sidereal time to corresponding mean time by Art. 17.

27. Reference has been made to the American Nautical Almanac, and rules given for taking out some required quantities. There are other quantities frequently required in Nautical Astronomy, such as

Declination of sun, moon, and planets; Equation of time, Semi-diameter, Horizontal Parallax of moon, etc.

In general it is necessary to take out the required quantities for the nearest Greenwich time to the given time, and interpolate in either direction to the given instant of Greenwich time.

Hourly differences are given to facilitate this work. As, however, the hourly differences themselves change quite materially in some cases, it may be found necessary to use *second differences*.

Formulæ have been given to meet each particular case. The author has found that in general they are of no practical assistance to the student, and even, in some cases, confusing. One thing may, however, be advantageously impressed upon the student, and that is, that *almost invariably it is necessary to first obtain the Greenwich time before consulting the Almanac.* At sea this is found from the chronometer, and on shore either by chronometer, or by applying to the local time of the place the longitude. *When the Greenwich time is apparent time, quantities pertaining to the true sun must be interpolated from Page I. of the month. When the time is mean time, then from Page II.* Quantities pertaining to other bodies are invariably given for the Greenwich mean time, excepting the negative R. A. of mean sun, which is given for the instant of Greenwich sidereal noon.

NOTATION FOR FOLLOWING CHAPTERS.

$L =$ latitude

$d =$ declination

$t =$ hour angle

$p =$ polar distance $= 90° - d$

$z =$ true zenith distance $= 90° - h$

$z' =$ apparent zenith distance $= 90° - h'$

$h =$ true altitude

$h' =$ apparent altitude

$Z =$ azimuth

$A =$ amplitude $= 90° - Z$

$q =$ position angle, or angle at the body.

CHAPTER V.

LATITUDE.

1. Latitude is the angular distance of a place on the surface of the earth, *north* or *south* of the equator.

As the celestial equator is in the same plane as the equator, and celestial meridians in the same planes with corresponding terrestrial meridians, it is evident that the zenith of an observer is the same angular distance from the celestial equator that his place is from the terrestrial equator. Distance *north* or *south* from the celestial equator is called *declination.* Hence the declination of an observer's zenith is equal to his Latitude.

2. To find the latitude from the altitude of any heavenly body on the meridian, the Greenwich time of the observation being known.

The observed altitude must be changed to true altitude, by applying errors of instrument, semi-diameter (if limb of body be observed), dip. parallax, and refraction. This is necessary in all observations, and, hereafter, when altitude is mentioned, it is to be considered as true altitude.

In Fig. 9, let $Z\,H\,N\,Q$ be a projection on the plane of the meridian of observer.

$E\,Q$ its intersection with plane of equator.

$I\,H$ its intersection with plane of true horizon.

$P\,P'$ the prolongation of the axis of the earth.

For the body X on the meridian, we have

$$EZ = L = ZX + EX = Z + d = 90° - h + d$$

for the body X'

$$EZ = L = ZX' - EX' = Z - d = 90° - h - d.$$

These are the two cases where d is north and south, or $+$ and $-$, and less than the latitude,

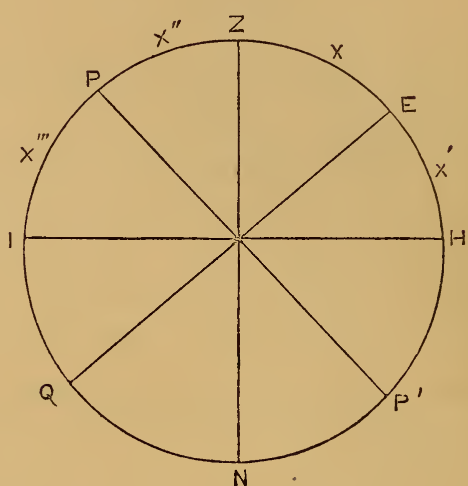
For the body X'',

$$L = d - Z = d - (90° - h)$$

for the body X''',

$$L = h + p.$$

FIG. 9.



3. Practical Navigators, in order that they may find the latitude instantly upon observation of the sun upon the meridian, make use of the following forms :

1*st*. When latitude and dec. are of same name, we have

$$L = 90° - h + d \text{ and } h = h' + \text{corr.}$$
$$L = 90° - (h' + \text{corr}) + d$$
$$L = (90° + d - \text{corr}) - h'$$

The portion within parentheses can be computed before the observation. All that remains to be done is to subtract observed altitudes, which may be done mentally.

2*d*. When lat. and dec. are of different names

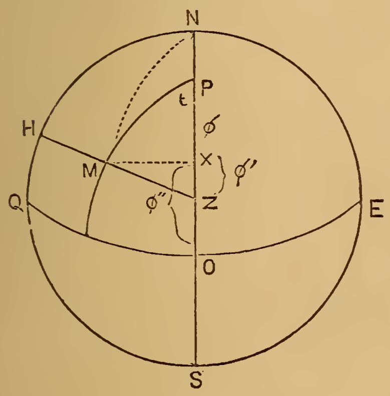
$$L = 90° - h - d$$
$$L = 90° - (h' + \text{corr}) - d$$
$$L = (90° - d - \text{corr}) - h'$$

In same way the portion within parentheses may be computed previous to the observation.

4. To find the latitude from an observed altitude of any heavenly body, at any time, the Greenwich time of the observation being given.

The declination of the body is found from the Greenwich time. The altitude corrected and hour angle of the body found, we then have,

FIG. 10.



$PM = p = 90° - d$

$ZM = Z = 90° - h$ and

$MPZ = t =$ hour angle given, to find

$PZ = 90° - L$

Let fall the perpendicular MX, and $L = 90° - (\phi + \phi')$ and if $\phi'' = 90° - \phi$, $\phi'' =$ the declination of foot of perpendicular, and as perpendicular may fall without the triangle

$$L = \phi'' \mp \phi'$$

$$\text{Cos } t = \tan d \cot \phi''$$

$$\text{Tan } \phi'' = \tan d \sec t \qquad (1.)$$

$$\text{Sin } d : \sin h = \sin \phi'' : \cos \phi'$$

$$\text{Cos } \phi' = \frac{\sin \phi'' \sin h}{\sin d} \qquad (2.)$$

which afford the solution. ϕ' when the perpendicular falls within the triangle is negative.

5. When the body is on the prime vertical, the perpendicular will fall near Z and $\phi' = 0$ nearly. When, therefore, the body is *near* the prime vertical, ϕ' becomes very small, and cannot be determined accurately by its cosine.

ϕ'' is marked N or S like the declination, and is in same quadrant as t, as the sign of its tangent in (1) is dependent upon that of sec. t. When $t > 6\text{h}$, $\phi'' > 90°$.

When the body has no declination, the perpendicular falls at

O, and $\phi'' = 0$, $L = \phi'$. When d is nearly 0, (1) approaches the undeterminate form. There are two values of L in the equation,

$$L = \phi'' + \phi', \text{ but unless } \phi'$$

be very small, the one may be selected which coincides most nearly to the supposed latitude. When ϕ' is less than 12 hrs. use 7 – place tables.

6. To find the effect of an error in the altitude we have

$$\text{Cos } \phi' = \frac{\sin \phi'' \sin h}{\sin d} \tag{2.}$$

Differentiating

$$-\sin \phi' \, d\,\phi' = \frac{\sin \varphi'' \cos h}{\sin d} \, d\,h.$$

$$d\,\phi' = -\frac{\sin \phi'' \cos h}{\sin d \sin \phi'} \, d\,h \tag{a.}$$

From (2)

$$\frac{\cos \phi'}{\sin h} = \frac{\sin \phi''}{\sin d}$$

which substituted in (a) gives

$$d\,\phi' = -\cot h \cot \phi' \, d\,h.$$

In triangle $Z M x$ of figure we have

$$\cos Z = \tan \phi' \tan h$$

hence

$$\sec Z = -\cot \phi' \cot h$$
$$d\,\phi' = d\,h \,.\, \sec Z,$$

substituting small finite differences.

$$\Delta\,\phi' = \Delta\,h \sec Z, \text{ nearly.}$$

$d\,\phi'$ is the *error* of ϕ' due to an error of h.

The *correction* to ϕ' for error of h would be

$$\Delta\,\phi' = -\Delta\,h \sec Z$$

When the body is on the meridian $Z = 0$, and numerically

$$\Delta\,\phi' = \Delta\,h.$$

The nearer Z is to 90° the greater will be $\Delta\,\phi'$.

7. To find the effect of an error in the time, or hour angle. We have

$$\text{Sin}\ h = \sin L \sin d + \cos L \cos d \cos t$$

$$0 = \cos L\ \text{d}\ L \sin d - \sin L\ \text{d}\ L \cos d \cos t - \cos L \cos d \sin t\ \text{d}\ t$$

$$d\ L = \frac{\cos L \cos d \sin t\ \text{d}\ t}{\cos L \sin d - \sin L \cos d \cos t} \qquad (a.)$$

In Fig. 10 we have in triangle $P\ N\ M$

$$\cos M\ N = \cos P\ N \cos P\ M + \sin P\ N \sin P\ M \cos N\ P\ M.$$

$$\cos N\ P\ M = -\cos t$$

and in triangle $N\ M\ P$

$$\cos M\ N = \cos L \sin d - \sin L \cos d \cos t$$

which substituted in (a) gives

$$\text{d}\ L = \frac{\cos L \cos d \sin t\ \text{d}\ t}{\cos M\ N} \qquad (b.)$$

and in $N\ Z\ M$

$$\cos M\ N = \cos N\ Z \cos Z\ M + \sin N\ Z \sin Z\ M \cos Z$$

$$\cos M\ N = \cos 90^\circ \sin h + \sin 90^\circ \cos h \cos Z$$

$$\cos M\ N = \cos h \cos Z$$

and (b) becomes

$$\text{d}\ L = \frac{\cos L \cos d \sin t\ \text{d}\ t}{\cos h \cos Z} \qquad (c.)$$

in triangle $P\ Z\ M$

$$\cos d : \cos h = \sin Z : \sin t$$

$$\sin t = -\frac{\cos h \sin Z}{\cos d}$$

In which Z is negative, being reckoned from meridian to the left.

Substituting in (c), reducing, and multiplying 2d term by 15 to reduce to arc,

$$\text{d}\ L = -15 \tan Z \cos L\ \text{d}\ t$$

The correction would be (substituting finite differences) nearly

$$\Delta\ L = -15 \tan Z \cos L\ \Delta\ t \qquad (d.)$$

When $Z = 0$, the effect of an error in time is 0.

When $Z = 90^\circ$, the effect of an error in time is incalculable.

In using this formula, be careful in correcting the time for the run between the observations for time and for latitude. Unless the time is accurately obtained the formula is of but little use.

8. To find the latitude when it is already approximately known.

$$\sin h = \sin L \sin d + \cos L \cos d \cos t$$

which by (39) and (139) reduces to

$$\cos (L-d) = \sin h + \cos L \cos d \text{ versin } t$$
$$L-d=z$$
$$\cos z_0 = \sin h_0 = \sin h + \cos L \cos d \text{ versin } t.$$

This is Bowd. 1st method for finding latitudes near noon. It is customary to use the latitude found by the "sailings" for the approximate latitude.

Table XXIII., Bowd., (latter part) contains the log. versin of t, with index increased by 5. As the second term has two members, a table of nat. sines and cosines will be necessary.

9. To find the latitude by two altitudes near noon when the time is not known.

The following method was devised by Prof. Chauvenet. The author has used it under different circumstances at sea, and strongly advises its substitution for the method in Art. 8, and also for the old method of circum.-merid. alts. Its use is restricted in the same manner as the method of *circum.-merid. alts.*

Its accuracy depends principally upon the precision with which the difference of alts. has been obtained.

As a preliminary to the method, we have, from Art. 8,

$$\sin h_0 - \sin h = 2 \cos L \cos d \sin^2 \tfrac{1}{2} t$$

by Trig (106)

$$\cos \tfrac{1}{2} (h_0 + h) \sin \tfrac{1}{2} (h_0 - h) = \cos L \cos d \sin^2 \tfrac{1}{2} t$$

But $h_0 = h$ nearly, and we may put

$$\cos \tfrac{1}{2} (h_0 + h) = \cos h_0 = \sin z_0 = \sin (L-d)$$

Hence

$$\sin \tfrac{1}{2} (h_0 - h) = \frac{\cos L \cos d \sin^2 \tfrac{1}{2} t}{(\sin L-d)} \qquad (a.)$$

Let $\Delta h = h_0 - h'$, the difference between the meridian and observed altitudes.

And as Δh and t are small

$$\sin \tfrac{1}{2} \Delta h = \tfrac{1}{2} \Delta h \sin 1''$$
$$\sin \tfrac{1}{2} t = \tfrac{1}{2} t \times 15 \sin 1''$$

(to express t in seconds of arc) substituting these in (a).

$$\Delta h = \frac{\cos L \cos d \, (\tfrac{1}{2} t \times 15 \sin 1'')^2}{\sin (L-d) \tfrac{1}{2} \sin 1''}$$

$$\Delta h = \frac{112.5 \sin 1'' \cos L \cos d}{\sin (L-d)} t^2$$

$$\sin 1'' = 0.000004848$$

$$\Delta h = \frac{0''.000545 \cos L \cos d}{\sin (L-d)} t^2$$

In this formula t is expressed in seconds. t is, however, usually expressed in minutes, and we must put $(60\, t)^2$ for t^2 and our equation becomes

$$\Delta h = \frac{1''.96349 \cos L \cos d}{\sin (L-d)} t^2$$

When $t = 1^m$

$$\Delta' h = \frac{1.96349 \cos L \cos d}{\sin (L-d)}$$

This equation may be computed for each value of L and d. Table XXXII., Bowd., contains the value of $\Delta' h$ for each 1° of declination from 0° to 24°, and each 1° of latitude from 0° to 70°, except when $(L-d) < 4°$.

We have

$$\Delta h = t^2 \Delta' h$$
$$h_0 = h + \Delta h, \text{ the meridian altitude.}$$

Let

h and h' = the true altitudes.

T and T', the corresponding hour angles in minutes of time.

$t = T' - T$, the difference of hour angles,

$T_0 = \tfrac{1}{2} (T' + T)$ the middle hour angle.

Then

$$\left. \begin{array}{l} h_0 = h + \Delta' h \, T^2 \\ h_0 = h' + \Delta' h \, T'^2 \end{array} \right\} (a.)$$

The mean of these equations is

$$h_0 = \tfrac{1}{2}(h + h') + \tfrac{1}{2}(T'^2 + T^2)\,\Delta' h \qquad (b.)$$

$$\tfrac{1}{2}(T'^2 + T^2) = \left(\frac{T' - T}{2}\right)^2 + \left(\frac{T' + T}{2}\right)^2 = (\tfrac{1}{2}\,t)^2 + T_0^2$$

which, substituted in (b), gives

$$h_0 = \tfrac{1}{2}(h + h') + (\tfrac{1}{4}\,t^2 + T_0^2)\,\Delta' h \qquad (c.)$$

The difference of equations (a) is

$$h - h' = (T'^2 - T^2)\,\Delta' h = 2\,T_0\,t\,\Delta' h,$$

hence

$$T_0 = \tfrac{1}{2}\,\frac{(h - h')}{t\,\Delta' h} = \frac{\tfrac{1}{4}(h - h')}{\tfrac{1}{2}\,t\,\Delta' h}$$

substituting this in (c), we have

$$h_0 = \tfrac{1}{2}(h + h') + (\tfrac{1}{2}\,t)^2\,\Delta' h + \frac{[\tfrac{1}{4}(h - h')]^2}{(\tfrac{1}{2}\,t)^2\Delta' h}$$

Hence the mean of the two altitudes, plus the square of one-half the interval between the observations multiplied by the change of altitude in one minute from noon (Table XXXII., Bowd.), plus the square of one-fourth the difference of altitude, divided by the first correction, is equal to the meridian altitude. The meridian altitude obtained may be proceeded with as usual.

10. To find the latitude from several altitudes taken near the meridian, the apparent times of observation being known.

See Bowd., page 202.

This method is commonly called the method of *circum-meridian altitudes.*

Let h', h'', h''', etc., be the several altitudes (observed)
t', t'', t''', etc., the corresponding hour angles.

We have for each reduction to the meridian from Art. 9,

$$\Delta_1 h = t'^2\,\Delta' h \;\therefore\; h_0 = h' + \Delta_1 h$$
$$\Delta_2 h = t''^2\,\Delta' h \;\therefore\; h_0 = h'' + \Delta_2 h$$
$$\text{etc.,} \qquad\qquad \text{etc.}$$

or

$$h_0 = \frac{h' + h'' + h''' + h_n}{n} + \frac{\Delta_1 h + \Delta_2 h + \Delta_n h}{n}$$

$$h_0 = \frac{h' + h'' + h_n}{n} + \frac{t'^2 + t''^2 + t^{n\,2}}{n} \Delta' h.$$

Hence the meridian altitude is equal to the mean of all the altitudes, plus the mean of the squares of the hour angles multiplied by the change of altitude in one minute from noon.

Table XXXIII., Bowd., contains the squares of hour angles up to 13m., and Table XXXII. the change in altitude in one minute from noon. When the heavenly body passes through or near the zenith, the change of altitude is too rapid for the assumption.

$$h_0 = h' + \Delta' h\,(T - x)^2$$
$$h_0 = h'' + \Delta' h\,T^2$$
$$h_0 = h''' + \Delta' h\,(T + x)^2$$

Subtracting the half sum of first and third equations from second, we deduce

$$\Delta' x^2 = h'' - \tfrac{1}{2}(h' + h''') \qquad (a.)$$

The difference of first and third gives

$$\Delta' h\,T = \frac{\frac{1}{4}(h' - h''')}{x}$$

$$\Delta' h\,T^2 = \frac{[\frac{1}{4}(h' - h''')]^2}{\Delta h' x^2}$$

which substituted in second equation, gives

$$h_0 = h'' + \frac{[\frac{1}{4}(h' - h''')]^2}{\Delta \frac{1}{2} x^2}$$

Substituting in this the value of $\Delta' h\,x^2$ from (a)

$$h_0 = h'' + \frac{[\frac{1}{4}(h' - h''')]^2}{h'' - \frac{1}{2}(h' + h''')}$$

which affords solution by giving h_0, the meridian altitude.

12. To find the latitude by the rate of change of altitude on prime vertical. (Prestel's Method.)

$$\sin h = \sin L \sin d + \cos L \cos d \cos t$$
$$\cos h \, \mathrm{d} h = -\cos L \cos d \sin t \, \mathrm{d} t$$
$$\mathrm{d} h = -\frac{\cos L \cos d \sin t \, \mathrm{d} t}{\cos h} \qquad (a.)$$

From the astronomical triangle we have

$$\cos d : \cos h = \sin Z : \sin t$$
$$\therefore \sin Z = \frac{\cos d \sin t}{\cos h}$$

which substituted in (a) gives

$$d h = -\cos L \sin Z \, d t.$$

Multiplying the second member by 15 to reduce to arc, changing sign for correction and transposing, we have

$$\Delta t = \frac{\Delta h}{15 \cos L \sin Z}$$

If now T' and T are respectively the hour angles of the altitudes h and h', we have for a small interval of time and small change of altitude

$$T' - T = t = \frac{h' - h}{15 \cos L \sin Z}$$
$$\cos L = \frac{h' - h}{15 \, t} \operatorname{cosec} Z \qquad (b.)$$

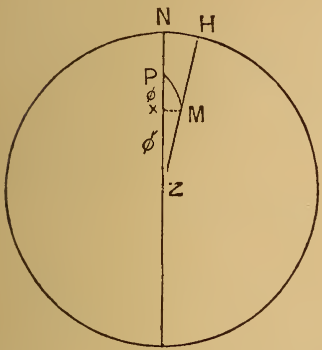
and when body is on prime vertical $Z = 90^\circ$ and

$$\cos L = \frac{h' - h}{15 \, t}$$

To use this observe two altitudes and note the times carefully. A very good approximate latitude may be obtained when the body is within 2° or 3° of the prime vertical. (b) may be used when Z is approximately known.

13. To find the latitude by an altitude of the Pole Star, the longitude of the place and local mean time being given.

Fig. 11.



In figure 11 let fall the perpendicular Mx, then in triangle $M\,x\,P$

$$\cos t = \cos p \tan \phi.$$

$$\tan \phi = \tan p \cos t$$

and as ϕ and p are small $(p = 1° \; 25')$

$$\phi = p \cos t \text{ nearly} \qquad (a.)$$

$$\phi = 90° - (L + \phi)$$

$$\cos p;\ \sin h = \cos \phi :\ (\sin L + \phi)$$

$$\sin h = \sin (L + \phi) \frac{\cos p}{\cos \phi}$$

but as p and ϕ are small, and their cosines nearly equal to 1, we have

$$\text{Sin } h = \sin (L + \phi)$$
$$h = L + \phi$$
$$L = h - \phi$$

When t is more than 6 hrs. and less than 18 hrs., $\cos t$ will be negative, and ϕ will be negative in (a), and numerically,

$$L = h + \phi$$

t is the hour angle of the star. The local time must be changed to sidereal time, and if $S =$ sid. time, then

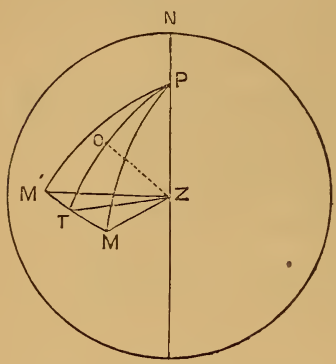
$$\phi = p \cos (S - {*}\text{'s R. A.})$$

If we consider p and star's R. A. to be constant, ϕ may be computed and tabulated for different values of S.

Owing to the changes of R. A. and dec. of Pole Star, such a table would require correction each year. It is better in practice to compute ϕ. Bowd. gives a table, page 206, for ϕ, but at the present time the table is incorrect.

15. To find the latitude by two altitudes with the elapsed time between them, supposing the declination to be the same at both observations, and the Greenwich time approximately known.

FIG. 12.



In the Fig. 12, let M and M' be the two positions of the body.

$h = 90° - Z\,M$, first altitude.

$h' = 90° - Z\,M'$, second altitude.

d = the declination common to each of the triangles.

$t = M\,P\,M'$, the difference of the hour angle; the elapsed apparent time in case of the sun; the elapsed sid. time in case of a star.

In the case of observations of a star the elapsed mean time noted by a watch or chronometer can be changed to a sid. time interval. In the case of the moon, the elapsed mean time can be corrected for the change in R. A. of moon during the interval.

If T = hour angle of body at M, and
T' = " " " M'
$t = T' - T$

We have the above given.

Let T be the middle point of $M\,M'$
Let $A = M\,T = M'\,T = \frac{1}{2}\,M\,M'$
$B = 90° - P\;T$, the declination of T,
$H = 90° - Z\;T$, the altitude of T.
$q = P\;T\,Z$, the position angle of T.

By assumption $P\;T\,M$ and $P\;T\,M'$ are equal right triangles, and angles $P\;T\,M$, $P\;T\,M' = 90°$, hence $q = 90° - Z\;T\,M = Z\;T\,M' - 90°$.

In the triangle $P\;T\,M$, by Nap. Rules, we have

$$\left.\begin{array}{l}\text{Sin } A = \cos d \sin \tfrac{1}{2}\,t \\ \text{Sin } B = \sin d \sec A\end{array}\right\} \quad (a.)$$

$$\text{Tan } B = \tan d \sec \tfrac{1}{2}\,t. \qquad (b.)$$

by which A and B may be found.

In the triangles $Z\ M\ T$, $Z\ M'\ T$ by Spher. Trig. (4)

$$\left.\begin{array}{l}\operatorname{Sin} h = \sin H \cos A + \cos H \sin A \sin q \\ \operatorname{Sin} h' = \sin H \cos A - \cos H \sin A \sin q\end{array}\right\} (c.)$$

The half difference and half sum of these

$$\operatorname{Sin} \tfrac{1}{2} (h - h') \cos \tfrac{1}{2} (h + h') = \cos H \sin A \sin q$$
$$\operatorname{Cos} \tfrac{1}{2} (h - h') \sin \tfrac{1}{2} (h + h') = \sin H \cos A$$

from which

$$\operatorname{Sin} H = \frac{\cos \frac{1}{2} (h - h') \sin \frac{1}{2} (h + h')}{\cos A} \qquad (d.)$$

$$\operatorname{Sin} q = \frac{\sin \frac{1}{2} (h - h') \cos \frac{1}{2} (h + h')}{\cos H \sin A} \qquad (e.)$$

which gives $90° - Z\ T$ and the angle $P\ T\ Z$.

We now have in the triangle $P\ T\ Z$

$$Z\ T = 90° - H$$
$$q = P\ T\ Z$$
$$B = 90° - P\ T$$

given, to find $P\ Z = 90 - L$.

Let fall the perpendicular $Z\ O$ and represent it by C.

Let $T\ O = Z$.

In triangle $Z\ O\ T$ by Nap. Rules

$$\left.\begin{array}{l}\cos q = \tan Z \tan \mathrm{H} \\ \tan Z = \cot H \cos q\end{array}\right\} \qquad (f.)$$

$$\left.\begin{array}{l}\sin H = \cos Z \cos C \\ \cos C = \sin H \sec Z\end{array}\right\} \qquad (g)$$

which determines Z and C.

$P\ O = 90° - (B \pm Z). - Z$ when perp. falls without the triangle $P\ T\ Z$.

In triangle $P\ O\ Z$

$$\sin L = \cos C \sin (B \pm Z) \qquad (h.)$$

In order to simplify the solution of the whole work, it will be necessary to find the values of C and Z, if possible, by using first data. To do this, we have in triangle $Z\ O\ T$:

$$\sin C = \cos H \sin q$$
$$\cos H = \frac{\sin C}{\sin q}$$

substituting this in (e) we have

$$\sin C = \frac{\sin \frac{1}{2}(h-h')\cos\frac{1}{2}(h+h')}{\sin A} \qquad (i.)$$

and substituting first of (g) in (d) gives

$$\cos Z = \frac{\cos \frac{1}{2}(h-h')\sin\frac{1}{2}(h+h')}{\cos A \cos C} \qquad (j.)$$

The values of C and Z thus obtained may be substituted in (h) and latitude found.

To condense the formula, and taking reciprocals of equations (a) and (b) we have

$$\left.\begin{array}{l} \text{cosec } A = \sec d \cos \frac{1}{2} t \\ \text{cosec } B = \text{cosec } d \cos A \\ \sin C = \sin \frac{1}{2}(h-h)\cos\frac{1}{2}(h+h')\text{ cosec } A \\ \sec Z = \sec \frac{1}{2}(h-h')\text{ cosec }\frac{1}{2}(h+h')\cos A \cos C \\ \sin L = \cos C \sin (B \pm Z) \end{array}\right\} \quad (k)$$

It is unnecessary to take out A and C from the tables, as the log. cos A may be taken corresponding to log. cosec A, and log. cos C corresponding to log. sin C.

The equations given above, give the form of Bowd. First method. They can be further simplified by finding B by its tan. in (b) and we may use

$$\left.\begin{array}{l} \tan B = \tan d \sec \frac{1}{2} t \\ \sin C = \sin \frac{1}{2}(h-h')\cos\frac{1}{2}(h+h')\sec d \text{ cosec } \frac{1}{2} t \\ \sec Z = \sec \frac{1}{2}(h-h')\text{ cosec }\frac{1}{2}(h+h')\sin d \text{ cosec } B \cos C \\ \sin L = \cos C \sin (B \pm Z) \end{array}\right\} \quad (l)$$

A, B, C, Z and L, are each numerically less than 90°. A is in 1st quadrant.

C is $+$ when 1st alt. is the greater, $-$ when the smaller. It really makes no difference about C, if we keep it less than 90°, as only its cosine is used. B has the same name as the declination.

16. We have seen that Z may be plus or minus according as the perpendicular $Z\ O$ falls without or within the triangle. By reference to the figure it will be seen that the perpendicular can fall without the triangle only when the continuation $M'\ M$ crosses the meridian between P and Z.

Hence the rule: mark Z north or south according as the zenith and elevated pole (N or S) are on the same side of the great circle, forming the two positions of the body. (See Bowd. p. 181.)

In the figure $P\ Z\ M > P\ Z\ M'$ and Z is $+$ or has same name as the latitude.

Hence, when the greater azimuth corresponds to the greater altitude, Z has the same name as the latitude.

By projecting a figure with perpendicular $Z\ O$ falling without the triangle $P\ T\ Z$, we would see that the greater azimuth corresponds to the lesser altitude, and we have: When the greater azimuth corresponds to the lesser altitude, Z has a different name from the latitude.

As Z is determined by its secant it cannot be accurately determined when very small. This will be the case when the altitudes are very great; when M and M' are near the prime vertical; or, in general, when the differences of the azimuths of M and M' are very small or nearly equal to 180°.

In the case of the sun this will be when the latitude and declination are nearly equal. This method cannot, therefore, be used with accuracy, when the sun crosses meridian near the zenith.

17. To find the latitude (circumstances as in last problem) **with an assumed latitude.** (Douwe's method. Bowd. 2d method)

In figure of last example

Let L' = the assumed latitude.

$T_0 = \frac{1}{2}(T' + T) = Z\ P\ T$ the middle hour angle.

$\frac{1}{2}t = \frac{1}{2}(T' - T)$ = half difference of hour angles.

From the second of (l), Art. 15, we have

$$\sin C = \frac{\sin \frac{1}{2}(h - h') \cos \frac{1}{2}(h + h')}{\cos d \sin \frac{1}{2} t} \qquad (a.)$$

and in triangle $P\ O\ Z$

$$\sin T_0 = \frac{\sin C}{\cos L}$$

$$T = T_0 - \tfrac{1}{2}t,\ T' = T_0 + \tfrac{1}{2}t$$

and (see Art. 8)

$$\cos z_0 = \sin h + 2 \cos d \cos L' \sin^2 \tfrac{1}{2} T$$
$$\cos z_0 = \sin h' + 2 \cos d \cos L' \sin^2 \tfrac{1}{2} T'$$

In place of $2 \sin^2 \frac{1}{2} T$, $2 \sin^2 \frac{1}{2} T'$ we may use versin T, and versin T'.

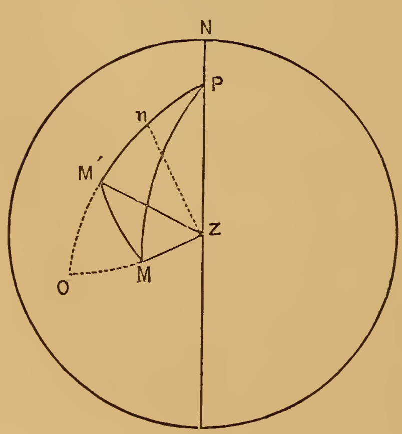
The latitude obtained by "Sailings," may be used, and should the latitude obtained differ largely from the assumed latitude, the work may be gone over again with the new latitude.

For computing (a) the first part of Tab. XXIII., Bowd., may be used conveniently.

18. To find the latitude from two altitudes of different bodies, or of same body when the change of declination is considerable, the Greenwich times being known.

Reduce the observed altitudes to true altitudes; the difference between the correct chronometer times must be taken, and when different bodies have been observed this interval changed to sid. interval. From this data compute T and T' the hour angles of the two bodies.

Fig. 13.



Given in Fig. 13:

$$d = 90° - P\ M$$
$$d' = 90° - P\ M'$$
$$h = 90° - Z\ M$$
$$h' = 90° - Z\ M'$$
$$T = Z\ P\ M$$
$$T' = Z\ P\ M'$$
$$t = T' - T = M\ P\ M'$$

Let fall the perpendicular $M\ O$, and represent declination of O by D'

$$P\ O = 90° - D'$$

and we have

$$\cos t = \tan d \cot D'$$

$$\tan D' = \tan d \sec t \qquad (a.)$$

$M'\ O = d' - D'$ and, representing $M\ M'$ by B

$\sin d$; $\cos B = \sin D'$; $\cos (d' - D')$

$$\cos B = \frac{\sin d \cos (d' - D')}{\sin D} \qquad (b.)$$

letting $P = O\ M'\ M$, and P' its supplement $M\ M'\ P$

$$\cos D' : \sin (d' - D') = \cot t : \cot P$$

$$\cos D' : \sin (D' - d') = \cot t : \cot P'$$

$$\cot P' = \frac{\cot t \sin (D' - d')}{\cos D'} \qquad (c.)$$

In the triangle $Z M' M$ calling the angle $Z M' M$, Q', we have

$$\sin \tfrac{1}{2} Q' = \sqrt{\frac{\cos \frac{1}{2} (B + h' + h) \sin \frac{1}{2} (B + h' - h)}{\cos h' \sin B}} \qquad (d.)$$

and if $q' =$ position angle $P\ M'\ Z$

$$q' = P' - Q' \qquad (e.)$$

In the triangle $P\ M'\ Z$, letting fall the perpendicular $Z\ n$, and representing $M'\ n$ by N', we have

$$\cos q' = \tan h' \tan N'$$

$$\tan N' = \cot h' \cos q'$$

$$P\ n = 90° - (d' + n')$$

and in the two triangles we have

$$\sin h' : \sin L = \cos N' : \sin (d' + N')$$

$$\sin L = \frac{\sin h' \sin (d' + N')}{\cos N'} \qquad (g.)$$

In (b) if the perpendicular $M\ O$ falls within the triangle, $M'\ O$ would be $= D' - d$ numerically.

The radical in (d) may have the positive or negative sign, and hence we may have two values of $q' = P' \mp Q'$.

In the figure

$$q' = P\ M'\ M - Z\ M'\ M.$$

q' will equal $P\ M'\ M + Z\ M'\ M$ when the greater azimuth corresponds to the lowest altitude. The ambiguity may therefore be removed by noting the azimuths at each observation. The other unknown quantities may be determined by their proper sign by restricting t to positive values less than 12 hours.

19. The hour angle T' in the triangle $P\ M'\ Z$ may be found, and thence the longitude if the times have been noted by a chronometer regulated to Greenwich time.

We have in the triangle $Z\ M'\ n$ and $P\ n\ Z$

$$M'\ n = N'$$
$$P\ n = 90° - (N' + d')$$
$$\cos q' = \tan N' \tan h'$$
$$\tan N' = \cot h' \cos q'$$
$$\sin N' : \cos (N' + d') = \cot q' : \cot T'$$
$$\cot T' = \frac{\cot q' \cos (N' + d')}{\sin N'}$$

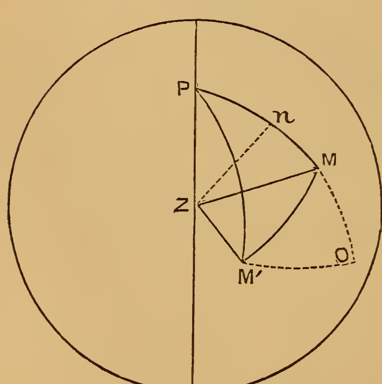
If L has been already found, we have also

$$\cos h' : \cos L = \sin T' : \sin q'$$
$$\sin T' = \frac{\sin q' \cos h'}{\cos L}$$

Sin T' and sin q' are positive when M′ is west of meridian ; negative when it is east.

20. In Arts. 18 and 19, we have employed the angles at M' in the triangle $P\ M'\ Z$. If in the accompanying Fig. 14, we had employed the angle M, and considered t positive in the direction opposite the diurnal rotation, remembering that q is less than 180° east of meridian, and greater than 180° west of the meridian, we should have

FIG. 14.



$$\tan D = \tan d' \sec t$$
$$\cos B = \frac{\sin d' \cos (d - D)}{\sin D}$$
$$\cot P = \frac{\text{Cot}\, t \sin (D - d)}{\cos D}$$

$$\sin \tfrac{1}{2} Q = \sqrt{\frac{\cos \tfrac{1}{2}(B + h + h') \sin \tfrac{1}{2}(B + h - h')}{\cos h \sin B}}$$

$$q = P \mp Q$$

$$\tan N = \cot h \cos q$$

$$\sin L = \frac{\sin h \sin (N + d)}{\cos N}$$

$$\cot T = \frac{\cot q \cos (N + d)}{\sin N}$$

The above formulæ may be deduced directly from the figure in the same manner as those of Art. 18.

21. If in the equations of Art. 20 we put

$D = -A$	$B = C$	$P = 90° - F$
$Q = Z$	$q = 90° - G$	$N = I$

we will have

$$\tan A = -\tan d' \sec t$$

$$\cos C = -\frac{\sin d' \cos (A + d)}{\sin A}$$

$$\cot F = -\frac{\tan t \cos A}{\sin (A + d)}$$

$$\sin \tfrac{1}{2} Z = \sqrt{\frac{\cos \tfrac{1}{2}(C + h + h') \sin \tfrac{1}{2}(C + h - h')}{\cos h \sin C}}$$

$$G = F \pm Z$$

$$\tan I = \cot h \sin G$$

$$\sin L = \frac{\sin d \sin (d + I)}{\cos I}$$

(See Bowd. 4th Method.

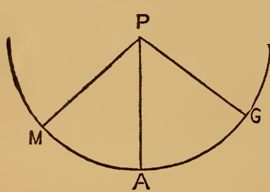
CHAPTER VI.

LONGITUDE.

1. Longitude is the angle at the pole between the meridian of the place and the prime meridian. In general the Greenwich meridian is taken as the prime.

In the Fig. 15, let $P\ G$ be the meridian of Greenwich (celestial) and $P\ A$ the meridian of any place west of it. $A\ P\ G$ would be the longitude of A. If now, $G\ P\ M$ be the hour angle of any heavenly body at Greenwich, $A\ P\ M$ is the hour angle of the same body at A, and $= G\ P\ M - A\ P\ M = A\ P\ G$.

Fig. 15.



Hence the difference in the hour angles of the same body at two meridians is equal to the difference of longitude, and if one of the meridians be that of Greenwich, is equal to the longitude.

If the place, A, be *east* of Greenwich, the angle $A\ P\ M > G\ P\ M$; the difference would still be the longitude east, or —

In order then to obtain the longitude at sea, it is necessary to determine the hour angle of some heavenly body at the same instant, at the meridian of the place and at Greenwich. The local hour angles of heavenly bodies are found by computation. The Greenwich hour angles are found indirectly by means of the chronometer.

2. The chronometer is a time measurer. A chronometer is called a *Greenwich chronometer* when it is regulated to Greenwich mean time. When we say *regulated* to Greenwich mean time, we mean that the reading of the chronometer, plus or minus a known correction, is the Greenwich mean time. In order to find this correction, we must know the *error* of the chronometer on some given day, and its daily *rate*.

The *error* of a chronometer is the amount that the chronometer is slow or fast at a given time.

The *rate* of a chronometer is the amount that it gains or loses daily.

It is evident that if we have then the *error* of a chronometer on some given date, and wish to find it on some other date, we must multiply the *rate* by the interval in days (and if necessary, decimal parts of days) and apply the result to the given error, according as the chronometer is gaining or losing, and also according as the date on which error is required is previous to or after the date on which the error is given.

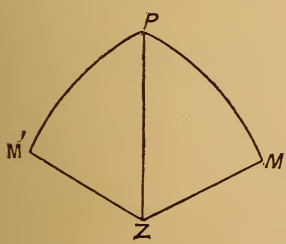
3. **To find the rate of a chronometer**, it is only necessary to know or find its error on different days; the difference in errors divided by the elapsed number of days, giving the rate.

4. **The chronometer correction** is the quantity which must be applied to the face of the chronometer to obtain the correct time. If the chronometer is slow the correction is +, if fast, correction is −

5. **To find the correction for a Greenwich chronometer by equal altitudes of the sun.**

In the case of a fixed star, the mean between the time of two equal altitudes is the time of transit. This may be compared with the computed time of transit and error of timepiece deduced.

FIG. 16.



In the case of the sun, owing to the change in declination, equal altitudes of M and M' do not give equal hour angles. The first observation is, however, taken at M, when the body has a certain declination, and the angle MPZ is not changed by the change of the declination from M to the meridian. In the two triangles, ZPM, ZPM' to find the error in t due to change in d,

We have

$$\sin h = \sin L \sin d + \cos L \cos d \cos t$$
$$0 = \sin L \cos d \, \mathrm{d}d - \cos L \sin d \cos t \, \mathrm{d}d - \cos L \cos d \sin t \, \mathrm{d}t$$

$$\mathrm{d}t = \frac{\sin L \cos d\, \mathrm{d}d - \cos L \sin d \cos t\, \mathrm{d}t}{15 \cos L \cos d \sin \mathrm{t}}$$

$$\mathrm{d}t = \frac{\tan L\, \mathrm{d}d}{15 \sin t} - \frac{\tan d\, \mathrm{d}d}{15 \tan t}$$

which gives the error in t due to change of d.

We may put, as the change of declination $\Delta\, d$ is small,

$$\Delta t = \frac{\tan L \Delta\, d}{15 \sin t} - \frac{\tan d \Delta d}{15 \tan t}$$

If $\Delta' d$ be the hourly difference of d given in the Ephemeris, and t the hour angle be expressed in hours

$$\Delta\, t = \frac{\tan L\, t \Delta' d}{15 \sin t} - \frac{\tan d\ t\ \Delta' d}{15 \tan t} \qquad (a.)$$

This gives an approximate expression for the error of t.

The correction to t would be

$$\Delta t = -\frac{\tan L\ t\ \Delta' d}{15 \sin t} + \frac{\tan d\ t\ \Delta' d}{15 \tan t} \qquad (b.)$$

If now the sun be observed at M and M' and the times noted by Greenwich chronometer, *the middle chronometer time* is the mean of the noted times. If the elapsed time is $2\,t$, the middle chronometer time would be

$$T + t$$

or

$$T' - t$$

This middle chronometer would be in error of time of transit by $\Delta\, t$ found above, and we should have for *chronometer time of apparent noon*

$$T + t + \Delta\, t, \text{ or}$$
$$T' - t + \Delta\, \mathrm{t}, \text{ or}$$
$$\frac{T + T'}{2} + \Delta\, t$$

If the first observation had been $P\ M$ it would be necessary to find the chronometer time of apparent midnight. By a similar process to the above, paying attention to the signs, we would have

$$\Delta t = \frac{\tan L \, t \, \Delta' d}{15 \sin t} + \frac{\tan d \, t \, \Delta' d}{15 \tan t} \qquad (c.)$$

6. If in (b) we put

$$A = -\frac{t}{15 \sin t} \text{ and } B = \frac{t}{15 \tan t}$$

we will have

$$\Delta t = A \, \Delta' d \tan L + B \, \Delta' d \tan d$$

L and d are $+$ when *north*, $-$ when *south*.

Δd and $\Delta' d$ are $+$ when the change of the sun's declination is towards the *north*,$-$when towards the *south*.

A is $-$ since t is < 12 hours.
B is $+$ when t is < 6 hours$-$when $t >$ 6 hours.

A and B may be computed for different values of t, and their logarithms tabulated. Such tables are given in "Chauvenet's method of finding the error and rate of a chronometer." The argument is $2\,t$, or the elapsed time.

In the equation for the lower branch of meridian the sign of A is changed as in (c).

$2\,t$ should be properly the elapsed apparent time. The interval is so small that this is generally neglected and the elapsed mean time used. It may be also corrected for the supposed or known rate of the chronometer. $\Delta' d$ is taken from the Nautical Almanac for the instant of apparent noon or apparent midnight.

In observing equal altitudes, use equal intervals of 10′ or 20′. It is not necessary that the altitudes be correct, but only that they should be the same on each side of the meridian. Use therefore, the same instruments at both observations, and be especially careful to use the same end of the roof of artificial horizon.

7. $\frac{T + T}{2} + \Delta t$ gives the chronometer time of apparent noon or of apparent midnight.

By applying the equation of time we have the chronometer time of *mean noon*, the difference between which and the longitude is the chronometer correction. If the correction of the chronometer to local mean time is required, we have only to omit the application of the longitude.

8. To find the correction to a Greenwich chronometer by a single altitude of any heavenly body.

As before, the observation must be taken at some place whose latitude and longitude are well determined.

We will have, therefore, in the astronomical triangle, the case when three sides are given to find the angle t, the formulæ for which are (Spher. Trig. 164, 165, 166).

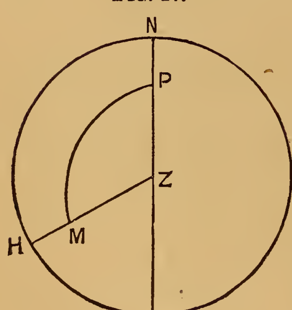
$$\sin \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}}$$

$$\cos \tfrac{1}{2} \mathrm{A} = \sqrt{\frac{\sin s \sin (s-a)}{\sin b \sin c}}$$

$$\tan \tfrac{1}{4} \mathrm{A} = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}}$$

We have given in Fig. 17

FIG. 17.



$$PM = p = 90° - d$$
$$ZM = z = 90° - h$$
$$PZ = \text{co}\, L = 90° - L, \text{ to find}$$
$$ZPM = t$$

The chronometer times of the altitudes are taken and their mean plus the supposed chronometer correction, gives us the Greenwich time, with sufficient accuracy for determining the declination of the body. The mean of the altitudes is taken and used as a single altitude. For finding t by its sine, we have, using the sides of the triangle directly,

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\sin \tfrac{1}{2} (p + z - \text{co}\, L) \sin \tfrac{1}{2} (z + \text{co}\, L - p)}{\cos L \sin p}} \qquad (a.)$$

It has been found more convenient to use the following values of the sides, viz.:

$90° - L$, $90° - h$, and p which gives

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\cos \tfrac{1}{2} (L + p + h) \sin \tfrac{1}{2} (L + p - h)}{\cos L \sin p}}$$

or, if we put

$$s' = \tfrac{1}{2}(L + p + h)$$

$$\sin \tfrac{1}{2} t = \sqrt{\frac{\cos s' \sin (s' - h)}{\cos L \sin p}} \qquad (b.)$$

Which is Bowd. formula, p. 209.

To determine t by its cos, using direct values of the sides, we would have

$$\cos \tfrac{1}{2} t = \sqrt{\frac{\sin \frac{1}{2}(\text{co } L + p + z) \sin \frac{1}{2}(\text{co } L + p - z)}{\cos L \sin p}}$$

or, if

$$s'' = \tfrac{1}{2}(\text{co } L + p + z)$$

$$\cos \tfrac{1}{2} t = \sqrt{\frac{\sin s'' \sin (s'' - z)}{\cos L \sin p}} \qquad (c.)$$

To determine t by its tangent, using direct values of the sides, we would have

$$\tan \tfrac{1}{2} t = \sqrt{\frac{\sin (s''' - \text{co } L) \sin (s''' - p)}{\sin s''' \sin (s''' - z)}}$$

in which

$$s''' = \tfrac{1}{2}(\text{co } L + z + p)$$

t is —when the body is east of meridian.

In case the sun is observed, if P. M., t is the L. A. T ; if A. M., it is 12 hrs. – L. A. T.

Bowd. Tab. XXVII. contains the direct value of t in P. M. column, and also 12 hrs. – t in A. M. column. In case any other heavenly body is observed, t is its hour angle, + when west and — when east of meridian. The L. M. T. may be found, and thence the Greenwich time by the method in the Chapter on time.

In the case of the sun, the L. A. T. is changed to mean time, and by applying the longitude, to Greenwich mean time. The mean chronometer time is compared with this to find the chronometer correction.

When $t < 6$ hrs. $\frac{1}{2} t < 45°$ and is better determined by b, as the sines of angles less than 45° vary more rapidly than the cosines. (See Chauv. Trig. Art. 112.)

When great precision is required, t is better determined by d, b and c are the most convenient formulæ for finding L. M. T. at sea. Many Navigators determine the errors and rates of their chronometers by *single altitudes.* It is advisable then to use (*d*).

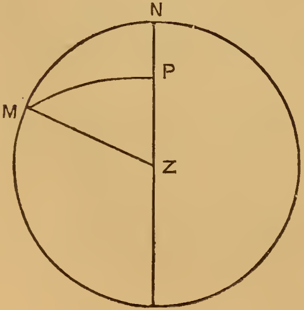
In taking observations for single altitudes take half the observations with each end of the roof. The times may be noted by a watch compared with the chronometer. If the interval between the comparison and observations is long, or the rate of the watch considerable, the watch times must be corrected for this change.

9. The two methods given are the only convenient methods which the Navigator can use with the instruments at his disposal for finding the correction for his chronometer. There are many ports where *time-balls* are dropped at the same instant each day for the convenience of the shipping in the harbor. Unless, however, they are dropped by electricity from some respectable observatory they are not to be depended upon.

10. As before stated, the methods of finding longitude at sea depend upon finding difference of time. The Greenwich chronometer, carefully regulated, furnishes the Navigator with the Greenwich time. The local time is found by observation of some heavenly body. The most common method is by (*b*) and (*c*) in Art. 8. Other methods are given. The latitude is found by applying the run of the ship to the latitude found at noon or by some other observation. The declination is taken from the Nautical Almanac for the Greenwich mean time, as shown by chronometer.

11. To find the hour angle (and thence the local time) of a heavenly body just visible in the horizon.

Fig. 18.



Let M be the body

$$P\,M = p = 90° - d$$

$$P\,N = L$$

In the triangle $M\,P\,N$ (Fig. 18), right angled at N, we have

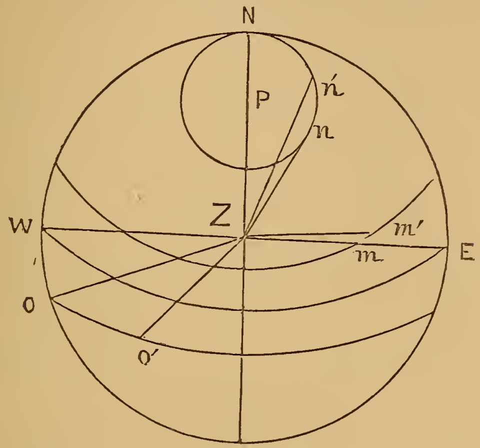
$$\cos M\,P\,N = \tan P\,N \cot P\,M$$

$$\cos M\,P\,N = -\cos t = \tan L \tan d$$

$$\cos t = -\tan L \tan d.$$

12. **To find the hour angle of a heavenly body when on or nearest to the prime vertical.**

FIG. 19.



In the case of the body at n, $d > L$, the body is nearest to the prime vertical when $Z\ n$ is tangent to its diurnal circle, and $P\ n\ Z = 90°$. We then have

$$\cos t = \tan p \tan L = \cot d \tan L.$$

If $d < L$ and of same name, as for a body whose path is $m\ m'$, the body will be on prime vertical at m and $P\ Z\ m = 90°$, and

$$\cos t = \cot L \tan d.$$

If d and L are of different names, as in the case of the body whose diurnal path is $o\ o'$, the nearest visible point to the prime vertical is in the horizon, and the solution is effected by the equation

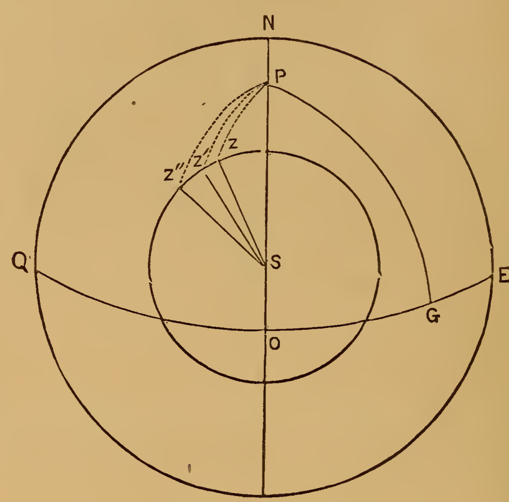
$$\cot t = - \tan L \tan d, \text{ of Art. 11.}$$

13. As A. M. and P. M. sights are enjoined in the directions of the Navy Department, it would be well if Navigators used the same altitudes for both observations. The corrections to the observed altitudes would be the same, and generally the longitudes determined would be at nearly equal intervals from noon.

They could each be reduced to the noon longitude, and their mean taken.

14. To find the longitude at sea by the intersection of circles of position.

Fig. 20.



At any instant of time the sun is in the zenith of some place whose latitude is equal to the declination of the sun at that instant, and whose longitude is equal to the Greenwich apparent time.

In Fig. 20, $P\ G$ is the meridian of Greenwich. $P\ O$ is the meridian of the place which has the sun S in its zenith. $S\ O$ is the latitude of this place and is equal to the declination of the sun. $G\ P\ S$ is the Greenwich apparent time, and is equal to the longitude of S.

If now any number of observers Z, Z', Z'', etc., situated on the circle, observe the sun at the same instant of Greenwich apparent time $G\ P\ S$, their zenith distances $Z\ S$, $Z'\ S$, $Z''\ S$ are equal. Such a circle is called a *circle of equal altitudes.*

Their Greenwich times being equal, they would each obtain from the N. A. the same declination $S\ O$. Each observer would have in the astronomical triangles $S\ P\ Z$, $S\ P\ Z'$, $S\ P\ Z''$, etc., the side $P\ S = p = 90° - d$ common, and the sides $S\ Z$, $S\ Z'$, etc., $= 90° - h$ equal. The hour angle of the sun at Z is $Z\ P\ S$, at Z', $Z'\ P\ S$, and at Z'', $Z''\ P\ S$. The difference in the values of these hour angles must be due to the different values of the third sides, $P\ Z$, $P\ Z'$, $P\ Z''$, etc. These sides are $90° - L$, $90° - L'$, $90° - L''$, etc. Hence the different values of the latitudes cause the different values of the hour angles, and thence the different values of the longitudes $G\ P\ Z$, $G\ P\ Z'$, $G\ P\ Z''$, etc.

An infinite number of *circles of equal altitude* may be drawn about S possessing the same properties as those described. If, therefore, with the sun as a centre, a circle be drawn upon a globe, all points upon this circle will have the same altitudes of the sun at the same instant.

As, therefore, the Greenwich time and altitudes are constant for any particular circle, an observer at Z, by using his own altitude and Greenwich time, and *assuming* the latitudes of Z, Z', Z'', etc., can determine the corresponding longitudes $G\ P\ Z$, $G\ P\ Z'$, $G\ P\ Z''$, etc.

Suppose an observer at Z', his latitude *unknown*, with his zenith distance $Z'\ S$, polar distance $P\ S$, and the assumed latitudes of Z and Z'' should determine their corresponding longitudes.

These assumed latitudes and determined longitudes may be plotted upon a globe, and the arc $Z\ Z''$ of the circle of equal altitudes drawn through them. The observer has a line $Z\ Z''$ upon which his position Z' is known to be. Such a line is called *a line of position.*

The direction of this line at any point is the direction of the tangent to the curve at this point. The direction of this tangent will be at right angles to the bearing of the sun at that point. *Hence by a line of position we may determine the azimuth of the sun.*

If now the observer wait until the sun has changed its bearing $n°$, and with the new values of the altitude and declination of the sun, and same values of the latitude, compute again a portion of the new circle of equal altitudes, as he is also on the second line of position, he must be on the intersection of the two. If this be plotted as before upon the globe, the intersection will be at Z', the latitude and longitude of which may be taken from the globe. To plot these curves accurately would require a larger globe than would be convenient. They may, however, be plotted upon a Mercator's chart.

By reference to the principles of construction of the Mercator's chart, it will be seen that only the loxodromic curve plots as a straight line. The circle of equal altitudes would plot as an irregular figure, its greatest diameter coinciding with the arc of the meridian $N\,O$. The whole figure could not indeed be plotted upon ordinary charts, unless the zenith distances $Z\,S$, $Z\,S'$, etc., were very small.

It is customary to plot only the small portion of the curve lying between the assumed latitudes, as that is all that is required. For small differences of latitude this would be practically a straight line. If the difference of latitude be great, or the chart a large scale one, latitudes between Z and Z'' may be assumed, the corresponding longitudes found and plotted, and the curve traced by hand through them.

In the practical use of this problem at sea, it is customary to assume latitudes 10′ or 20′ on each side of the supposed one, and determine the corresponding longitudes.

In general, assume the latitudes to cover any supposed error of the latitude.

In the foregoing, the discussion has been confined to the sun. The body S may be any other heavenly body which can be conveniently observed.

15. If, between the observations, the observer should change his position, as is generally the case at sea, the first observation may be corrected to the position of the second by correcting the altitude, or, more conveniently, by moving the first line of position to the place of the second observation.

If the first observation be taken at Z (Fig. 21), and the ship run to Z', $Z'\,O'$ or $Z\,O$ is the correction to the altitude, or zenith distance $Z\,S$, to find the zenith distance $Z'\,S$ at the same instant.

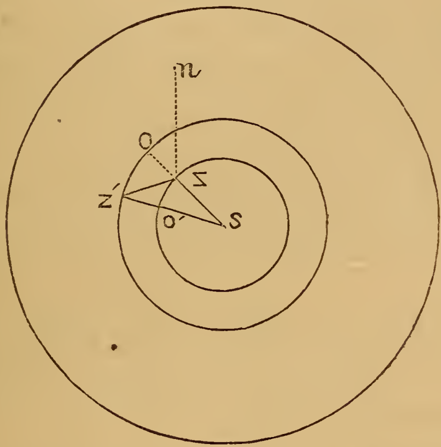
If the distance be small, $Z\,O'$ may be considered as a right line having the direction of the tangent at Z, and

$$Z'\,O' = Z\,O = \varDelta\,z = Z\,Z' \sin Z\,Z'\,O' = Z\,Z' \sin Z'\,Z\,O'$$
$$N\,Z\,Z' = C = \text{course}$$
$$N\,Z\,O = 180° - Z$$
$$\varDelta\,z = -\varDelta\,h = Z\,Z' \cos\left[C - (180° - Z)\right]$$
$$\varDelta\,h = -Z\,Z' \cos\,(C - Z)$$

If the first observation be at Z' in the same way we will have

$$\varDelta\,h = Z\,Z' \cos\,(C - Z)$$

FIG. 21.



The difference between C and Z is taken, and Z reckoned from the north point 180°. Then, if the difference between C and Z is $< 90°$, $\Delta\,h$ is additive; if $> 90°$, $\Delta\,h$ is subtractive. (See Bowd. Rule, p. 183.) The equation for $\Delta\,h$ may be solved by the Traverse Table. Find $(C–Z)$ at top or bottom of page, and the distance sailed in distance column, opposite in difference of latitude column, is the correction in minutes and tenths to be added to altitude when difference is less than 90°. If the difference is greater than 90°, find $180°—(C–Z)$, as before, and correction is subtractive.

To move the line of position, lay off on the chart the distance $Z\,Z'$ in the direction of the course sailed between the observations, through the extremity draw the line $Z' O$ parallel to $Z O'$. This evidently accomplishes the same result as correcting the altitudes. It possesses the advantage of being simple, and when the chart has the magnetic compass plotted upon it, the compass course can be laid off between the observations.

The method of correcting the altitude must be used, however, in the case of Double Altitudes for Latitude.

We have seen that the line of position is at right angles to the bearing of the sun. If the sun is on the prime vertical at both observations, the lines of position will run north and south, and there will be no intersection.

If $L = d$ nearly, the lines of position will not change their direction sufficiently to depend upon their intersection.

When the body is near the prime vertical, errors in the latitudes have the least effect upon the corresponding longitudes.

When the body is near the meridian, errors in the longitude have their least effect upon corresponding latitudes.

Latitudes may be assumed, and the corresponding local mean times found, or the longitudes may be assumed, and the corresponding latitudes determined by Art. 4, Chap. on Latitude.

16. If there is an uncertainty in the altitude, draw on each side of the line of position lines parallel to it, and distant from it, the amount of the supposed uncertainty, and the position will be somewhere within this belt.

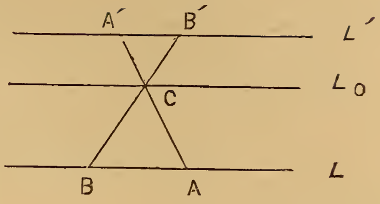
In the same manner, if there is an uncertainty in the Greenwich time, parallels may be drawn upon each side of the line of position equal to this uncertainty.

17. Near the coast, when charts are on a sufficient scale, there is no difficulty in determining the position with a considerable degree of accuracy. At long distances from the coast line, our charts are generally upon too small a scale to admit of an accurate plotting of the lines. This may be remedied best by projecting upon a piece of paper a sectional chart which shall cover the difference of latitude and longitude.

The latitude may be found by computation, as follows:

Let l_1 l_2 the longitudes of A and B in latitude L.
l_1' l_2' the longitudes of A' and B' in latitude L'
L_0 = latitude of C.

FIG. 22.



From the similarity of the triangles $A\ B\ C$ and $A'\ B'\ C$

$$l_2'\ l_1' : l_1 - l_2 = B'\ C : B\ C$$

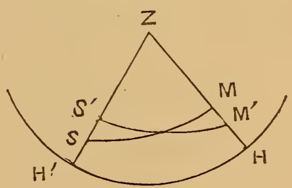
$$(l_2' - l :) + (l_1 - l_2) : l_1 - l_2 = B\ B' : B\ C = L' - L : L_0 - L$$

$$L_0 = L + \frac{(L^1 - L)\ (l_1 - l_2)}{(l'_2 - l_1) + (l_1 - l_2)}$$

The Navigator will find the method of Art. 17 preferable to this. It does not require great nicety in the construction of the chart. The latitude and longitude of the intersection may be transferred from this chart to the one in use.

18. To find the longitude by means of observed lunar distances. (See Vol. II., No. 4, of the Ast. Journal. Chauv. Method.)

FIG. 23.



The observation is supposed to give the apparent distance and apparent altitudes of the two objects; but if the latter cannot be observed, they must, in order to apply the present method, be previously computed by known rules. Taking at once the most general case, namely, that in which the object observed with the moon also has parallax, let us take "the sun." The formulæ will require no change for a planet, and for a star no change beyond making the parallax zero.

Let, then in Fig. 23, Z being the zenith of the observer,

$d = S'\ M'$ = the apparent distances of moon's and sun's centres.

$h = M'\ H$ = the moon's apparent altitude.

$H = S'\ H'$ = the sun's apparent altitude.

d_1, h_1, H_1, the distance and altitudes referred to that point of the earth's axis which lies in the vertical of the observer, which point we shall distinguish as the point O.

We shall then have

$$\frac{\cos d_1 - \sin h_1 \sin H_1}{\cos h_1 \cos H_1} = \frac{\cos d - \sin h \sin H}{\cos h \cos H}$$

and if

$$m = \frac{\sin h \sin H_1}{\sin h \sin H} \qquad n = \frac{\cos h_1 \cos H_1}{\cos h \cos H_1}$$

then

$$\cos d - \cos d_1 = (1 - n) \cos d - (m - n) \sin h \sin \mathrm{H} \qquad (1.)$$

Let

$$\Delta d = d_1 - d, \qquad \Delta h = h_1 - h, \qquad \Delta H = H - H_1,$$

then

$$\cos d - \cos d_1 = 2 \sin \tfrac{1}{2} \Delta d \sin (d + \tfrac{1}{2} \Delta d)$$

$$n = \frac{\cos (h + \Delta h) \cos (H - \Delta H)}{\cos h \cos H}$$

$$= \left(1 - \frac{2 \sin \frac{1}{2} \Delta h \sin (h + \frac{1}{2} \Delta h)}{\cos h}\right) \times \left(1 + \frac{2 \sin \frac{1}{2} \Delta H \sin (H - \frac{1}{2} \Delta H.}{\cos H}\right),$$

$$1 - n = \frac{2 \sin \frac{1}{2} \Delta h \sin (h + \frac{1}{2} \Delta h)}{\cos h} - \frac{2 \sin \frac{1}{2} \Delta H \sin (H - \frac{1}{2} \Delta H)}{\cos H}$$

$$+ \frac{4 \sin \frac{1}{2} \Delta h \sin \frac{1}{2} \Delta H \sin (h + \frac{1}{2} \Delta h) \sin H - \frac{1}{2} \Delta H)}{\cos h \cos H}$$

also observing the relations

$$\sin h_1 \cos h = \tfrac{1}{2} [\sin (2 h + \Delta h) + \sin \Delta h]$$

$$\cos h_1 \sin h = \tfrac{1}{2} [\sin (2 h + \Delta h) - \sin \Delta h]$$

$$\sin H_1 \cos H = \tfrac{1}{2} [\sin (2 H - \Delta H) - \sin \Delta H]$$

$$\cos H_1 \sin H = \tfrac{1}{2} [\sin (2 H - \Delta H) + \sin \Delta H]$$

we find

$$m - n = \frac{\sin h_1 \cos h \sin H_1 \cos H - \cos h_1 \sin h \cos H_1 \sin H}{\sin h \cos h \sin H \cos H}$$

$$= \frac{\sin \Delta h \sin (2 H - \Delta H) - \sin \Delta H \sin (2 h + \Delta h)}{2 \sin h \cos h \sin H \cos H}$$

if then we put

$$A_1 = \frac{2 \sin \frac{1}{2} \Delta h \sin (h + \frac{1}{2} \Delta h) \cos d}{\cos h}$$

$$B_1 = -\frac{\sin \Delta h \sin 2 (H - \Delta H)}{2 \cos h \cos H}$$

$$C_1 = -\frac{2 \sin \frac{1}{2} \Delta H \sin (H - \frac{1}{2} \Delta H) \cos d}{2 \cos H}$$

$$D_1 = \frac{\sin H \sin (2h + \Delta h)}{2 \cos h \cos H}$$

the equation (1) becomes

$$2 \sin \tfrac{1}{2} \Delta d \sin (d + \tfrac{1}{2} \Delta d) = A_1 + B_1 + C_1 + D_1 - A_1 C_1 \sec d. \qquad (2.)$$

This rigorous formula may be adapted for practical use in several ways requiring auxiliary tables. I proceed to give the transformation which appears to require the fewest and simplest tables.

If the terms of (2) are reduced to seconds, we shall have

$$\Delta d \sin (d + \tfrac{1}{2} \Delta d) = A_1 + B_1 + C_1 + D_1 - A_1 C_1 \sin 1'' \sec d. \qquad (3.)$$

in which

$$A_1 = \frac{\Delta h}{\cos h} . \sin (h + \tfrac{1}{2} \Delta h) \cos d$$

$$B_1 = -\frac{\Delta h}{\cos h} . \frac{\sin (2H - \Delta H)}{2 \cos H}$$

$$C_1 = -\frac{\Delta H}{\cos H} . \sin (H - \tfrac{1}{2} \Delta H) \cos d$$

$$D_1 = \frac{\Delta h}{\cos H} . \frac{\sin (2h + \Delta h)}{2 \cos h}$$

Let

p = moon's horizontal parallax reduced to the point O.
r = moon's refraction.

P, R, the same quantities for the sun, then

$$\Delta h = p \cos (h - r) - r$$
$$\Delta H = R - P \cos (H - R).$$

The neglect of R in the term $P \cos (H - R)$ produces an error altogether inappreciable in practice; but the error produced by omitting r in the term $p \cos (h-r)$ may amount to $1''$, and we shall therefore take

$$\cos (h - r) = \cos h + \sin r \sin h$$

$$\varDelta h = p \cos h - r + p \sin r \sin h$$

$$= (p \cos - r)\left(1 + \frac{p \sin r \sin h}{p \cos h - r}\right)$$

If we develop the last term, and put

$$k = r \tan h,$$

we shall have, designating the term by K,

$$K' = \frac{p \sin r \sin h}{p \cos h - r} = k \sin 1'' \left(1 + \frac{k}{p \sin p}\right)$$

in which p may be taken at its mean value; and since k and h decrease together, it will be found that K is nearly constant, its maximum being .000296, and its minimum .000285. A wider range will be admitted if we allow for the variations of the barometer and thermometer, and of p; but without here entering into more details, it will suffice to state that the error of the value

$$K = .00029$$

is always less than .00006 so long as $h > 5°$, and the formula

$$\varDelta h = (p \cos h - r)(1 + K)$$

gives $\varDelta h$ within $0''.05$ at a mean state of air, and within $0''.2$ in all cases.

Let now

$$r^1 = \frac{r}{\cos h}, \qquad R^1 = \frac{R}{\cos H}.$$

The quantities r^1 and R^1 will be given by a "Refraction Table for Lunars," which with the argument *apparent altitude* will give the refraction divided by the cosine of the altitude, and will be arranged precisely like the ordinary tables of refractions. The corrections for the barometer and thermometer may be arranged as usual in nautical tables, with the arguments *height of barometer (or thermometer)* and apparent altitude; or, which is preferable, with the refraction itself instead of the altitude, for with the latter arrangement the same table will serve to give the correction either of r or of r^1. These quantities then being substituted, the corrections of the apparent altitudes become

$$\varDelta h = (p - r^1)\,(1 + K)\cos h$$
$$\varDelta h = (R^1 - P)\cos H$$

and the terms of (3) become

$$A_1 = (p - r)\,(1 + K)\sin\,(h + \tfrac{1}{2}\,\varDelta\,h)\cos d$$
$$B_1 = -\,(p - r^1)\,(1 + K)\,\frac{\sin\,(2\,H - \varDelta\,H)}{2\cos H}$$
$$C_1 = -\,(R^1 - P)\sin\,(H - \tfrac{1}{2}\,\varDelta\,H)\cos d$$
$$D_1 = (R^1 - P)\,\frac{\sin\,(2\,h + \varDelta\,h)}{2\cos h}$$

The term $A_1\ C_1 \sin 1'' \sec d$ is very small, its maximum being only about 1″. It is easy to obtain an approximate expression for it, and to combine it with the term A_1; for in so small a term we may take

$$C_1 = -\,R'\sin H\cos d = -\,k'\cos d$$

where $k' = R\tan H$; and without sensible error in most cases we may take $k'\sin 1'' = K$, so that

$$C_1\sin 1''\sec d = -\,K$$

and

$$A_1 - A_1\,C_1\sin 1''\sec d = (p - r')\,(1 + K)^2\sin\,(h + \tfrac{1}{2}\,\varDelta\,h)\cos d.$$

The error of this evaluation of the term $A_1\ C_1 \sin 1'' \sec d$ is produced chiefly by the neglect of P, and is therefore appreciable only in the case of the planet Venus. If we suppose the extreme case in which P, $p - r'$, and H are all at their maximum values, the error in this term is

$$0''.44\cos d$$

and since the equation (3) is yet to be divided by $\sin d$, the final error in the distance is

$$0''.44\cot d$$

and can amount to 1″ only when $d < 24°$. Moreover, the error is of less importance in the case of Venus, because much less than the probable error of observation arising from an imperfect *bisection* of the planet's disc in the feeble telescope of the sextant.

Now let

$$\left.\begin{aligned}
A &= (1+K)^2 \,.\, \frac{\sin\,(h+\frac{1}{2}\,\varDelta\,h)}{\sin\,h} \\
B &= (1+K) \,.\, \frac{\sin\,(2\,H-\varDelta\,H}{\sin\,2\,H} \\
C &= \frac{\sin\,(H-\frac{1}{2}\,\varDelta\,H)}{\sin\,H} \\
D &= \frac{\sin\,(2\,h+\varDelta\,h)}{\sin\,2\,h} \\
A' &= (p\,-r')\,A\,\sin\,h\,\cot\,d \\
B' &= -\,(p\,-r)\,B\,\sin\,H\,\mathrm{cosec}\,d \\
C' &= -\,(R'-P)\,C\,\sin\,H\,\cot\,d \\
D' &= (R'-P)\,D\,\sin\,h\,\mathrm{cosec}\,d
\end{aligned}\right\}\ \text{(A.)}$$

then our formula (3) becomes

$$\varDelta\,d\,.\,\frac{\sin\,(d+\frac{1}{2}\,\varDelta\,d)}{\sin\,d}=A'+B'+C'+D'.$$

Developing the first number, it becomes

$$\varDelta\,d\left(1+\frac{2\,\sin\,\frac{1}{4}\,\varDelta\,d\,\cos\,(d+\frac{1}{4}\,\varDelta\,d)}{\sin\,d}\right)$$

so that if we put

$$x=-\,\frac{\varDelta\,d^2\,\sin\,1''\,\cos\,(d+\frac{1}{4}\,\varDelta\,d)}{\sin\,d}$$

or, with sufficient accuracy

$$x=-\,\varDelta\,d^2\,\sin\,1''\,\cot\,d \qquad \text{(B.)}$$

we have finally

$$\varDelta\,d=A'+B'+C'+D'+x \qquad \text{(C.)}$$

The logarithms of A, B, C and D can be given in extremely simple tables, requiring little or no interpolation, the arguments for log A and log D being $p-r'$ and h, and those for log B and log C being $R'-P$ and H. A, B, C and D may then be computed with the greatest ease. The value of x can be given in a small table with the arguments $\Delta\,d$ and d, the table being first entered with the approximate value of $\Delta\,d=A'+B'+C'+D$.

The advantages of the preceding processes are conceived to be—1st. The formula is almost rigorously exact, representing the correction of distance in all practical cases within 1″. 2d. The logarithmic computation is simple and brief. 3d. The tabulated logarithms require no correction for the height of the barometer and thermometer. In no one of the approximative methods in use are these features combined. Those which are based upon accurate formulas either require troublesome computations, or are shortened by the use of tables in which a mean refraction is used, and no ready method is given for correcting the logarithms in these tables for the actual state of the air. Such, for the most part, are Bowditch's methods. It would hardly be necessary to allude to those which are not based upon accurate formulas, were it not that one of this character has been adopted in a comparatively recent work of great merit in most respects, *Raper's Practice of Navigation.* The approximate method employed in that work is one received from *Mendoza Rios*, apparently without a very critical examination; in favorable circumstances, and particularly in low latitudes, it may be so applied as to be sufficiently accurate, but in high latitudes cases are common in which the error in the distance is 10′, and in the *extreme* case the error is 50″.

If we compare our method with the shortest of the rigorous processes of spherical trigonometry, we find—1st. It is simple in the logarithmic computation, requiring only four-decimal, or, at most, five-decimal logarithms. It is also an important simplification for the practical navigator, that the distance and altitudes are not required to be combined (to form, for example, their half sum, etc.) previously to referring to tables, as in almost every other method, approximative or rigorous. 2d. It separates the principal corrections for the moon and sun, the principal correction for the moon being $A'+B'$, and that for the sun being $C'+D'$. The advantage of this separation appears in the method to be given for computing the correction for contraction of the moon's and sun's semidiameters by refraction. (Section IV.)

3d. Correction for the Compression of the Earth.—In the preceding investigation d_1 h_1 H_1 represent the distance and altitudes referred to the point O. This reference may be made in the case of the moon by employing a horizontal parallax, equal to her equatorial horizontal parallax, increased in the ratio $\frac{a_1}{a}$, a denoting the equatorial radius of the earth, and a_1 the distance of the observer from the point O, which distance is the normal of the spheroid, and is expressed by

$$a_1 = \frac{a}{\sqrt{(1 - \varepsilon^2 \sin^2\phi)}}$$

Where ε = eccentricity of the meridian.
ϕ = geodetic latitude.

This process is subject to a slight theoretical error, the amount of which will presently be estimated.

If we denote by $-a\,i$ the distance from the centre of the earth to the point O, and put

π = moon's equatorial horizontal parallax.
ρ = distance of the moon from the centre of the earth.
δ = moon's geocentric declination.
d' = angular distance of the moon and sun referred to the centre of the earth,
π_1, ρ_1, δ_1, d_1, = the same quantities referred to point O,
Δ = sun's declination,
a = difference of right ascensions of the moon and sun, then we have the known formulas—

$$a\,i = \frac{a\,\varepsilon^2 \sin\phi}{\sqrt{(1 - \varepsilon^2 \sin^2\phi)}}$$

$$\left.\begin{aligned} \rho_1 \cos\delta_1 &= \rho\cos\delta \\ \rho_1 \sin\delta_1 &= \rho\sin\delta + a\,i \end{aligned}\right\} \qquad (4.)$$

whence, very nearly,

$$\rho_1 = \rho + a\,i\,\sin\delta$$

$$\sin\pi_1 = \frac{a_1}{\rho_1} = \frac{a_1}{\rho}\left(1 + \frac{a\,i\sin\delta}{\rho}\right) - 1$$

$$= \frac{a_1}{a}\sin\pi\,(1 - i\sin\pi\sin\delta + \text{etc.}$$

or, with extreme accuracy,

$$\pi_1 = \pi\,.\,\frac{a_1}{a} - \frac{\varepsilon^2\sin^2\pi\sin\phi\sin\delta}{\sin 1''}$$

The maximum value of the last term is only 0″.2, so that in the present application we may take

$$\pi = \pi \,.\, \frac{a_1}{a}$$

and the correction of π

$$\pi_1 - \pi = \pi \,.\, \frac{a_1 - a}{a}$$

may be given in a small table with the arguments ϕ and π. The similar correction of the sun's or a planet's parallax is insensible in practice.

If, then, in the computation of (A), (B), and (C), we employ for p the value $p = \pi_1$ we obtain d_1. To reduce finally to the centre of the earth, we have

$$\left.\begin{array}{l} \cos d' = \sin \varDelta \sin \delta + \cos \varDelta \cos \delta \cos a \\ \cos d' = \sin \varDelta \sin \delta_1 + \cos \varDelta \cos \delta_1 \cos a \end{array}\right\} \qquad (6.)$$

from which combined with (4) we find

$$\rho \cos d' - \rho_1 \cos d_1 = - a\, i \sin \varDelta$$

or by (5)

$$\cos d' - \cos d_1 = \frac{a\, i}{\rho} (\sin \delta \cos d_1 - \sin \varDelta)$$

$$2 \sin \tfrac{1}{2} (d' + d_1) \sin \tfrac{1}{2} (d' - d_1) = i \sin \pi \,(\sin \varDelta - \sin \delta \cos d_1)$$

and with great accuracy for our present purpose,

$$d' - d_1 = \frac{i\, \pi \sin \varDelta}{\sin d_1} - \frac{i\, \pi \sin \delta}{\mathrm{con}\; d_1} \qquad (D.)$$

a formula easily put into tables, especially if we employ a mean value of π, which will never produce an error of more than about 1″. If any one, however, desires to compute this correction directly, it may be done by the formula

$$d' - d_1 = N \pi \sin \phi \frac{\sin \varDelta}{\sin d_1} N \pi \sin \phi \,.\, \frac{\sin \delta}{\tan d_1} \qquad (D.)$$

in which

$$N = \frac{\varepsilon^2}{\sqrt{(1 - \varepsilon^2 \sin^2 \phi)}}$$

and we may employ without sensible error the value of N corresponding to $\phi = 45°$, or $\log N = 7.8170$, the compression being $\frac{1}{300}$.

The computation of this correction would be rendered at once simple and accurate in practice, if the ephemeris contained the log of

$$N = \frac{\sin \varDelta}{\sin d'} = \frac{\sin \delta}{\tan d} = N$$

(which is equivalent to a logarithm introduced by Bessel into his ephemeris for the same purpose), for we should then have

$$d' - d_1 = N' \sin \phi \qquad (7.)$$

4th. Corrections for the Contraction of the Moon's and Sun's semidiameters by Refraction.—The apparent distance of the centres of the moon and sun has been supposed above to have been found in the usual manner from the observed distance of the limbs, by adding the apparent semidiameters; or when the moon has been observed with a planet or star, by adding or subtracting the moon's semidiameter alone, according as her nearest or farthest limb has been observed. At low altitudes the elliptical figure of the disc must be taken into consideration; for the refraction being different at points of the limb which have different altitudes, the result is an apparent contraction of every semidiameter, the vertical ones being the most, and those perpendicular to the vertical the least contracted. It becomes necessary to obtain a general expression for the contraction of that semidiameter which lies in the direction of the distance, and makes an angle q with the vertical circle. If we put

s = horizontal semidiameter of the moon + the augmentation,
s_0 = the apparent vertical semidiameter
s' = " inclined "
$\varDelta s_0$ = contraction of vertical " $= s - s_0$
$\varDelta s'$ = " inclined " $= s - s'$
$\varDelta r$ = difference of refractions at the centre of the moon and the observed point on the limb,

we have nearly

$$\varDelta s = \varDelta r \cos q.$$

But the apparent altitude of the centre being h, $\varDelta s_0$ is the difference of refractions at the apparent altitudes h and $h + s_0$, while $\varDelta r$ is the difference of refractions at h and $h + s' \cos q$, whence

$$\Delta s_0 : \Delta r = s_0 : s' \cos q$$

$$\Delta r = \frac{s'}{s_0} . \Delta s_0 \cos q = \Delta s_0 \cos q \text{ (nearly)}$$

$$\Delta' s = \Delta s_0 \cos^2 q \qquad (8.)$$

a known formula which agrees very nearly with the hypothesis that the figure of the disc is an ellipse. It is evident, however, that the lower half of the disc is more flattened than the upper half; but if Δs_0 be taken as the mean of the contractions of the upper and lower vertical semidiameters, the preceding formula will be in error only 0″.4 at the altitude 10°, and 1″.2 at 5°; the maximum values of Δs_0 at those altitudes being respectively 10″ and 30″. The changes of the thermometer and barometer may also sensibly affect the value of Δs_0 at low altitudes, but only by 4″ in the improbable case of the *highest barometer and lowest thermometer*, and $h = 5°$. It will hardly be necessary to attend to this small error in practice; nevertheless, it can readily be done without any further reference to the refraction tables, for the computer will already have before him r'_0 the mean value of r', and $\Delta r'_0$ the sum of the corrections of r'_0 for barometer and thermometer; so that he may find at once the proportional correction of $\Delta s'$, which is

$$\Delta s' . \frac{\Delta r'}{r_0'} .$$

Now the angle q is given by the formula

$$\cos q = \frac{\sin H - \sin h \cos d}{\cos h \sin d}$$

and we have from the formula (A)

$$\frac{B'}{B\,(p - r') \cos h} = - \frac{\sin H}{\cos h \sin d}, \quad \frac{A'}{A\,(p - r') \cos h} = \frac{\sin h \cos d}{\cos h \sin d},$$

$$\cos q = \left(\frac{B'}{B} + \frac{A'}{A}\right) \frac{1}{(p - r') \cos h}.$$

If we assume $A = 1$, $B = 1$, we shall have

$$\cos q = - \frac{A' + B'}{(p - r') \cos h}$$

$$\Delta s' = \Delta s_0 . \frac{(A' + B')^2}{(p - r')^2 \cos^2 h} \qquad (E.)$$

which is easily put into tables. A table with the arguments h and $p - r$ may give the value of

$$\frac{\Delta s_0}{(p - r')^2 \cos^2 h}$$

and a second table with the arguments $A' + B'$ and "the number from the first table" may give $\Delta s'$.

In order to ascertain the degree of accuracy of the formula (E), we observe that the errors in cos q produced by taking $A = 1$, $B = 1$, are

$$e = (A - 1) \frac{\tan h}{\tan d}, \quad e' = (1 - B) \frac{\sin H}{\cos h \sin d};$$

the errors in $\cos^2 q$ are

$$2\, e \cos q \text{ and } 2\, e' \cos q$$

and the errors in $\Delta s'$ are therefore

$$e_1 = \frac{2\, \Delta s_0 (A-1) \tan h \cos q}{\tan d}, \quad e_1' = \frac{2\, \Delta s_0 (1-B) \sin H \cos q}{\cos h \sin d}$$

The greatest values of e_1 and e'_1 at different altitudes, are shown as follows, taking $\cos q = 0$, $H = 90°$, in order to represent the extreme cases:

h	$e_1 \tan d$	$e'_1 \sin d$
5°	0.45″	0.02″
10	0.16	0.00
15	.08	.00
30	.02	.00
50	.00	.00

It appears, therefore, that the error of the formula (E), like that of (8), becomes sensible only at those low altitudes where extreme precision is unattainable on account of the uncertainty of the refraction. We may therefore safely employ it as sufficiently accurate for all cases.

When the sun is observed with the moon, a similar correction must be applied to his semidiameter. If

$Q =$ angle at the sun,
$S =$ true semidiameter of the sun,
$S_0 =$ apparent vertical semidiameter of the sun,
$S' =$ " inclined " "
$\varDelta S_0 =$ contraction of vertical semidiameter $= S - S_0$,
$\varDelta S' =$ " inclined " $= S - S'$,

then as above

$$\varDelta S' = \varDelta S_0 \cos^2 Q$$

$$\cos Q = \frac{\sin h - \sin H \cos d}{\cos H \sin d} = \left(\frac{C'}{C} \frac{D'}{D}\right) \frac{1}{(R' - P) \cos H}$$

and assuming

$$C = 1, D = 1,$$

we have

$$\cos Q = \frac{C' + D'}{(R' - P) \cos H}$$

$$\varDelta S' = \varDelta S_0 \,.\, \frac{(C' + D')}{(R' - P)^2 \cos^2 H} \qquad (F.)$$

which is even more accurate than (E), and is put into tables in the same manner.

The corrections $\varDelta s'$ and $\varDelta S'$ should strictly be applied to the semidiameter, and should appear in the value of a employed in the computation $\varDelta d$; but since the values of A', B', C', and D' are required in finding $\varDelta s'$ and $\varDelta S'$, we have to employ a value of d which may in extreme cases be in error by about $30''$. This produces a small error in each of the terms A', B', C', D', which could in practice be eliminated only by repeating the computation with the corrected value of d. But this repetition is unnecessary, as the error in $\varDelta d$ is rarely more than $0.''5$; and it will suffer to apply $\varDelta s'$ and $\varDelta S'$ directly to d_1.

In order, however, to show generally the effect upon $\varDelta d$ of small errors in d, let us differentiate the equation (C), regarding the term x (of the second order) as constant, and taking $A = 1$, $B = 1$, $C = 1$, $D = 1$ (which also amounts to considering terms of the second order as constant). We find

$$\delta \varDelta d = - \frac{(h - r)(\sin h - \sin H \cos d) \sin 1'' \delta d}{\sin^2 d}$$

$$+\frac{(R'-P)\ (\sin H - \sin h \cos d) \sin 1''\ \delta\ d}{\sin {}^2 d}$$

$$\delta\, \Delta\, d = -[(p-r')\cos Q \cos H - (R'-P)\cos q \cos h]\,\frac{\sin 1''\,\delta\, d}{\sin d} \quad (9.)$$

This formula shows at once that the maximum of $\delta\ \Delta\ d$ occurs when the two bodies are in the same vertical circle, the moon being the higher body, for this condition gives cos q = —1, cos q = 1, so that the two terms obtain the same sign.

The following table shows the maximum effect upon $\Delta\ d$ of the error of 1″ in d, computed by formula (10), for the several values of h and H; the least value of $h - H\ (= d)$ being 20°.

H	*h*							
	25°	35°	45°	55°	65°	75°	85°	90°
° 5	″ 3.6	″ 2.4	″ 1.9	″ 1.5	″ 1.3	″ 1.2	″ 1.1	″ 1.1
15	..	3.2	2.2	1.7	1.4	1.2	1.1	1.0
25	..	..	2.9	2.0	1.5	1.3	1.1	1.0
35	..	..	..	2.6	1.8	1.4	1.1	1.0
45	..	..	..	..	2.2	1.5	1.2	1.0
55	..	..	..	..	..	1.8	1.2	1.0
65	..	..	..	..	..	..	1.3	0.9

and at the same time $p - r'$ and $R' - P$ have their greatest values. In this position, we have $d = h - H$, and the formula for the maximum of $\delta\ \Delta\ d$ is therefore

$$\delta_0\, \Delta\, d = -\ [(p - r')\cos H + (R' - P)\cos h]\,\frac{\sin 1''\,\delta\, d}{\sin (h - H)}.$$

This table of extreme errors shows clearly enough that the error arising from the neglect of $\Delta\ s'$ and $\Delta\ S'$ in the value of d employed in computing $\Delta\ d$, is too small to require any departure from the process already indicated. For the Navigator must bear in mind that all observations at very low altitudes are subject to two principal sources of error :—1st, the uncertainty of the refraction, which no process of calculation can eliminate ;

and 2d, the imperfect definition of the limb of the moon or sun in the vicinity of the horizon. If a method of computation involves only errors which in every case are less than these unavoidable errors, it satisfies the essential condition of a good method.

CHAPTER VII.

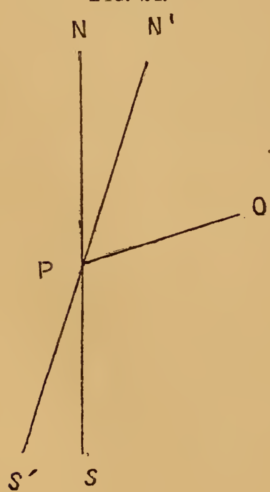
THE COMPASS.

1. A magnetized needle or bar of steel balanced and allowed to turn freely on a pivot, will take a position in a particular direction, which is called the *magnetic meridian.*

The direction in which the north end points is the *magnetic north.* It varies or declines from the true north differently at different places on the earth ; and even at the same place at different times. Delicate observations show a small diurnal fluctuation of a few minutes, also a progressive change or one of very long period,—on the Atlantic coast of the United States, of 2′ to 5′ westerly in one year.

2. If a circular card marked with the horizon points be attached to such a needle, its several points will deviate from the corresponding points of the horizon, all by the same amount and in the same direction.

FIG. 24.

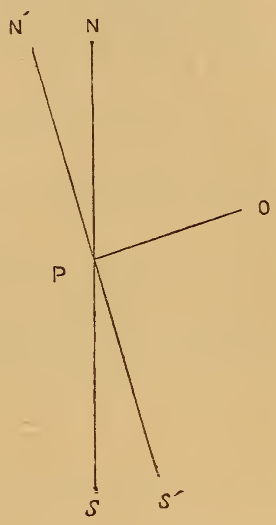


Let P be any place,
$N\ S$ its true meridian,
$N'\ S'$ its magnetic meridian,
$N\ P'\ N'$ is the variation.

In Fig. 24 it is east, $N'\ S'$ being to the right of $N\ S$.

3. The *magnetic declination, or variation of the compass*, at any place, is the angle which the magnetic meridian of that place makes with the true meridian. It is *east*, if the magnetic meridian is to the *right*; *west*, if the magnetic meridian is to the *left* of the true meridian.

FIG. 25.



In Fig. 25 it is west, $N' S'$ being to the left of $N S$.

The point of view, or position from which the observer is supposed to look, being at P.

4. Let

O be any object, terrestrial or celestial.

$P\ O$, its horizontal direction from P.

$A = N P O$, the true azimuth or bearing of O from the N point of the horizon.

$A' = N' P O$, the magnetic bearing of O from the magnetic north.

$D = N P N'$, the variation.

If towards the *right* be regarded as the *positive* direction of these angles, and towards the *left* as the *negative* direction, we have from both figures, and with O in any position,

$$N P N' = N P O - N' P O, \text{ or,}$$
$$D = A - A',$$

positive, or to the *right*, for Fig. 1;
negative, or to the left, for Fig. 2.

If A and A' denote bearings or angular distances from any other points than the true north and magnetic north, for instance the east or the west, we evidently still have—

$$D = A - A';$$

or, translated into common language:

The magnetic declination, or variation, at any place is equal to the difference of the true and the magnetic bearings of any object; it is east if the number or point which expresses the true bearing is to the right of the number or point which expresses the magnetic bearing; but west if to the left.*

5. From equation (1) we also have

$$A = A' + D;$$

or, the variation must be applied to a compass bearing (or course) to the right hand if east, to the left hand if west, in order to find the true bearing (or course).

6. The same equation also gives

$$A' = A - D;$$

or, the variation must be applied to a true bearing (or course) to the left hand if east, to the right hand if west, in order to find the compass bearing (or course).

7. To find the variation, it is necessary to determine both the true bearing and the magnetic bearing of some object; at the same instant if the object be in motion.

8. The true bearing or azimuth of a celestial object may be found—

First.—From an observation of its altitude (Prob. 1). This may be used to the best advantage when the azimuth changes most slowly with the altitude, *i. e.*, when a given change or supposed error of the altitude produces the least change of azimuth. The most favorable position of any object is when its azimuth is

* The numbers or points are supposed to be read from the same compass card, the observer looking at them from the centre.

nearest 90° ; the unavailable position is on the meridian. High altitudes and great declinations, especially if of a different name from the latitude of the place, are to be avoided.

Second.—When it is in the horizon, or its apparent altitude above the sea horizon is 33′+ the dip, (Prob.) its amplitude, or bearing from the east or west point of the horizon, is readily determined by the solution of a spherical right triangle ; or when the declination is less than 23° 28′, by Tab. VII. (Bowd.).

Third.—From the local time.

The most favorable time for a circumpolar star is that of its greatest elongation from the meridian ; for other objects, when they are on or near the prime vertical. A more exact knowledge of the time is requisite, when the observation is made near the time of meridian passage, especially at very high altitudes.

Fourth.—From the measurement with a theodolite, or other azimuth instrument, of the azimuth angle between the two positions of the body at the same altitude east and west of the meridian.

9. The true bearing of a terrestrial object at any point may be found, from the measurement—

First.—With a theodolite, or other azimuth instrument, of the horizontal angle ; or,

Second.—With a sextant, of the angular distance between the terrestrial object and some celestial object, whose azimuth at the same instant is found either from its altitude or the local time.*

It is not necessary to have two observers, or that the observations of altitudes and horizontal (or oblique) angles should be simultaneous. One observer may measure an altitude, then the horizontal (or oblique) angle, then another altitude, noting the time. On the supposition that the altitudes increase or decrease uniformly we have, *as the interval* of time between the observa-

* Sometimes called "an astronomical bearing."

tions for altitude, is to the interval between the first observation for altitude and the observation for hor. (or ob.) angle, so is the difference of altitude to the reduction of the first altitude.

Measurements of the horizontal (or oblique) angle may be made before and after the observation of altitude, and interpolated in the same way.

10. When precision is requisite, it is necessary to keep in mind—

First.—That a change of the point of observation of .001 of the distance of the terrestrial object may change its bearing more than 3′.4.

Second.—That the higher the altitude of an object, the more requisite is the careful adjustment of the instrument used in the measurement of the horizontal angle.

Third.—That greater care is requisite in the measurement of the direct angular distance, the greater the inclination to the horizon of the oblique plane which passes through the two objects; the *apparent* altitude, or angle of elevation, of the terrestrial object above the eye of the observer must also be determined.

Fourth.—That in measuring terrestrial angles with a sextant or circle of reflection, the axis about which the index moves is the proper centre of the instrument, and the reading should be increased by the parallactic angle, which is inversely as the distance of the *object seen direct.*

For a distance of 500 feet it is about 1′ in the common sextant. But it is combined with the index correction, if the observation for the latter be made with an object at the same or nearly the same distance.

These are important considerations in accurate surveys, and in making with precision meridian lines.

Ordinarily the sun is the most convenient celestial object. For use in connection with a compass, precision in the true bearing to the nearest 5′ is generally sufficient.

11. The magnetic bearing is observed directly with a compass. The two chief forms of this instrument are the *surveyor's compass*, in which the graduated circle revolves with the line of sight, while the reading points, which are the extremities of the needle, remain fixed;

And the *mariner's compass*, and in its more refined form, the *azimuth compass*, in which the graduated circle attached to the needle remains fixed, while the pointer revolves with the line of sight.

With the best surveyor's compasses a precision of 5′, or with the best azimuth compasses a precision of 10′, is rarely attainable.

12. To obtain even this degree of precision, it is necessary—

First.—To correct for the index-error of the instrument. This correction is the same for all bearings; and may be found for each compass (and compass-card) by bearings of a number of objects in different directions, whose true magnetic bearing has been determined by more delicate instruments. Once carefully found, it nay be marked as a *constant* correction.

If it is neglected, the bearings observed are "compass bearings," and the variation found is the *variation of that particular compass;* in distinction from the true magnetic bearings and the true magnetic declination.

Second.—To correct for eccentricity, or for the pivot not being in the centre of the graduated circle.

With the surveyor's compass this error is eliminated by opposite readings of the graduated circle.

Azimuth compasses are not sufficiently delicate for the refinements of this correction. But the maximum error may be found by measuring horizontal angles of about 90°, which have been measured by a more reliable instrument.

Third.—To attend to the balancing of the needle or compass card. Sealing wax dropped on that part of the card which requires depression is sometimes used.

As the north end of the needle dips or is depressed in north magnetic latitude, and the south end in south magnetic latitude, readjustment is generally necessary after a considerable change of latitude.

Fourth.—That the sight vane or vanes and their axis of rotation should be parallel, also perpendicular to the graduated circle, if there be one on the compass box.

Observations on a plumb line, or other well defined vertical line, made on the land, furnish a test of these adjustments.

Fifth.—That at the instant of observation the sight vanes should be vertical.

This is the more important the greater the elevation of the object.

Azimuth compasses are furnished with a mirror attached to the sight vane, so that objects of considerable elevation may be observed by reflection. This mirror should be perpendicular to the plane passing through the eye-vane and the thread of the sight-vane, to which the mirror is attached. This may be tested by observations on a well-defined vertical line on shore.

13. For ordinary sea purposes a precision of 30′, or even 1°, is sufficient. But even this requires some attention to the several points of the last article.

It is desirable that all compasses on board ship should be tested—those for steering as well as those for more delicate use, and their errors noted or adjusted, if of sufficient importance.

The error arising from the motion of the ship is less sensible if the plane of the gimbals coincide with that of the card (when the instrument is at rest), and pass through the point of the pivot. Generally, however, the pivot is placed above the gimbals and the card, since it is necessarily above the centre of gravity of the needle and its attachments.

14. Magnetic needles, when not suspended, should be put away in pairs, parallel, and with the north pole of one against the south pole of the other, and separated, either in different boxes or by a piece of cork or soft wood.

15. Small pieces of iron in the vicinity of a compass may produce a sensible deflection of the needle. Ships have often wandered far from their intended course from a few nails or a knife or other small iron article being carelessly placed in a binnacle.

If two compasses are near each other the north pole of one needle repels the north and attracts the south pole of the other. They will then be deflected, and both in the same direction (and equally if equal magnets), unless their direction from each other is N.E.S. or W. (magnetic). In some intermediate direction, near four points from the meridian, the deflection will be the greatest.

Hence the comparison of two compasses placed side by side is an imperfect test of their agreement or accuracy. When two binnacles are used they should be at least $4\frac{1}{2}$ feet apart. The disagreement of the compasses placed in them is, however, not wholly due to their influence upon each other, but to other sources of disturbance.

16. Electricity will disturb the needle. If the glass cover be rubbed with dry silk, a delicate compass may be rendered for the time useless. A strong electric current may weaken the magnetism of a needle, or even reverse its poles. Lightning may produce such a change.

17. On shore, in particular locations, very marked deviations of the needle are observed.

In ships, particularly those of iron, and in a less degree those which have iron as a part of their cargo or armament, there are peculiar causes of disturbance. The observations of Professor Airy show that a part of the iron is permanently magnetic, or nearly so, changing only very slowly, and that another portion is magnetic by induction, and varies with its position with reference to the meridian and in different magnetic latitudes.

A ship may be regarded as two assemblages of magnets, one permanent, the other variable; and each acting upon the compass in any particular position as a single magnet, whose force is the resultant of the combined forces of all its parts. The disturbance will be different in different parts of the ship. Observations have been instituted on board of some iron ships for determining the position where the compass is least disturbed.

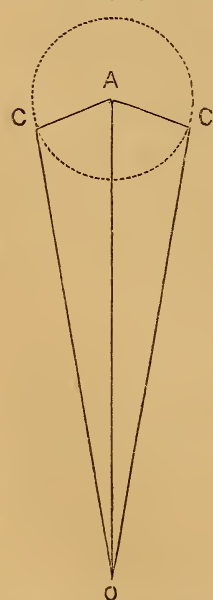
The standard compass on board some ships is placed between the binnacles, and elevated so as to command a view of the horizon, to affect less the steering compass, and to be farther above the level of the disturbing magnets.

18. The *deviation* of the compass produced by these local causes varies with the direction of the ship's head.

The resultant of the permanent magnet may be resolved into two forces: one tending to draw* the N. pole of the needle towards the ship's head, and having a maximum effect when the ship heads E. or W. by compass; the other tending to draw* it towards the starboard side, and having a maximum effect when the head is towards the N. or S. The variable magnet is regarded by Prof. Airy as having its maximum effect when the ship heads N.E., S.E., S.W., or N.W., by compass; but this may not always be the case.

19. To find the local deviation for different directions of the ship's head, it is necessary as the ship turns round—either by being swung round intentionally, or at sea in a calm, or with light baffling winds, or at anchor by the tides—to observe the bearing of some well-defined object as the head comes successively to each point of the compass. The direction of the ship's head should be carefully noted at the time of taking each bearing. It is well to note it by the binnacle compass as well as by that employed in the observations.† The compass must occupy the same position during the whole series of observations, as the local deviation determined is for that position only.

Fig. 26.



20. *If the object be terrestrial and so distant*, that the swinging of the ship produces no sensible change in its actual direction as seen from the position of the compass, no other observations are necessary.

To ascertain what this distance must be in a given case,

Let O be the object, CC' the extreme positions of the compass, as the ship swings round the point A.

* Or to repel it, in which case the effect is regarded as negative.

† The heading by other compasses in different parts of the ship may also be observed simultaneously.

Put $d = A\ C$;
$D = O\ A$, the distance of the object;
$O = A\ O\ C$, the parallactic angle.

We have

$$\sin O = \frac{d}{D}$$

or, since O is very small (in minutes),

$$O = \frac{d}{D \sin 1'} = 3438' \frac{d}{D}$$

If d is expressed in feet and D in sea miles,

$$O = \frac{d}{6087\ D \sin 1'} = O'.5648 \frac{d}{D}$$

whence

$$D = \frac{O' \times .5648\ d}{O}$$

Examples.

(1) $d = 300$ ft., $D = 6$ miles; then $O = 28'$.

(2) $d = 500$ ft., and it is desirable that O shall not exceed $30'$; then D must not be less than $\frac{O' \times .5648 \times 500}{30'}$ or 94 sea miles.

If the bearings have been taken as the ship headed at the intended points, that is at equal intervals round the compass, the mean of the whole series will be the true compass bearing; the difference of this mean from each observed bearing will be the local deviation for the corresponding direction of the ship's head, and should be marked east, if this mean bearing is to the right of the observed; west if the mean bearing is to the left of the observed. A table of the local deviations may be formed by writing in one column the direction of the ship's head, and in another the corresponding deviations.

Or, the "ship's head by compass" may be laid off on a straight line at proper intervals as *abscissas*, and the corresponding deviations as *ordinates*, and a curve drawn through the several points thus determined.

Or the differences of some conveniently assumed bearing from the observed bearings, may be laid off as ordinates, and a line drawn parallel to the axis, and so as to divide the curve symmetrically. The distance of the curve from this line at the several points will be the local deviation.

The scale for the ordinates may be greater than that for the abscissas.

This graphical method is more convenient when the bearings have not been taken as the ship headed at the intended points, but at unequal intervals; or if any have been omitted.

21. If *the object be near*, an observer may be stationed at it who will make observations at the same times that the bearings are taken on board the ship; the instants being indicated by some preconcerted signals made on board.

First.—With a *theodolite* carefully adjusted, and with its horizontal limb clamped, he may direct the telescope towards the position of the ship's compass, and read the instrument; or

Second.—With a sextant he may measure the horizontal angles between the ship's compass and some well-defined object, taking into account, when necessary, the angles of elevation of the two;* or

Third.—With a *plane-table* he may draw on paper lines in the direction of the ship's compass, and measure the angles which they make with some lines drawn at pleasure; or

Fourth.—With a good compass he may take reciprocal bearings.

By any of these instruments the changes in the direction of the ship's compass from the object (and as well, of the object from the compass) are directly measured. These observations, then, furnish the means of reducing the bearings observed on board to what they would have been if made at a fixed position, or upon an object whose direction was not varied.

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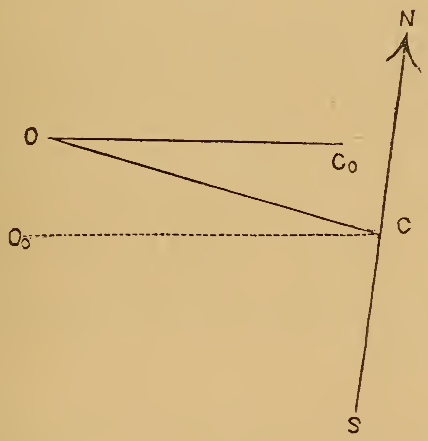
* Let A and A' be the two angles of elevation, then the horizontal or azimuth angle will be an angle of a spherical triangle, of which the two adjacent sides are $(90°—A)$ and $(90°—A')$, and the opposite side is the observed angular distance of the two objects.

Such fixed position, or rather its direction from the shore object, is entirely arbitrary. That the reductions may be small, and all applied in the same direction, and conveniently computed, let the assumed *zero line* of direction be that for which the shore instrument would read the smallest number of degrees noted.*

The several readings or angles measured by the shore instruments, diminished by this assumed number of degrees, are respectively the parallactic reductions to be applied to the corresponding bearings observed with the compass on board the ship. They are to be applied to the right when the zero line is to the right of the actual line of direction; to the left, when the zero line is to the left of the actual line of direction.

This precept is easily demonstrated:

FIG. 27.



Let O be the object.

C the position of the ship's compass.

C_o the position to which the bearings are to be reduced.

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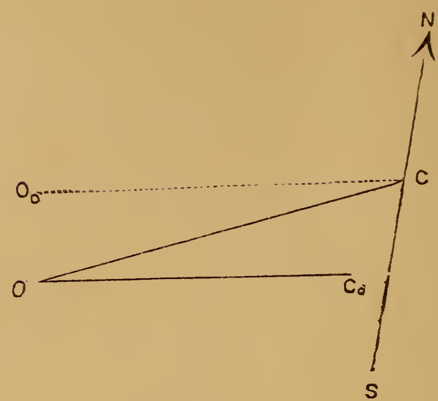
* If the readings are on different sides of a *zero-point*, the line of direction for that zero-point is most convenient; or the readings on one side may be increased by 180° or 360°.

FIG. 28.



CO is the line whose bearing is observed.

$C_0\,O$, parallel to $C\,O_0$, is the line whose bearing is required.

The reduction is the angle $O_0CO = C_0OC$.

In *Fig.* 27, OC_0 is to the right of OC, and the reduction is to be applied to the right.

In *Fig.* 28, OC_0 is to the left of $O\,C$, and the reduction is to be applied to the left.

This is evidently true, whatever may be the direction of the meridian line NS.

The bearings observed on board the ship having been thus reduced, they may be used as if they had been made on a very distant object, and the local deviations computed and tabulated, or plotted, as in Art. 20.*

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* If a good chart of the harbor on a scale sufficiently large is available, the position of the ship's compass at each observation may be found either by cross bearings on two distant objects, or by measuring with a sextant the horizontal angles between them; and plotted upon the chart. The magnetic bearings of the shore object may then be measured on the chart: the differences of these from the corresponding compass bearings will be the deviations.

In some harbors, poles or other well defined marks are placed so as to range with a distant object on particular magnetic bearings, as each 5° or 1 point. With such facilities, the observer on board has only to note the range, or between what two ranges, and where between, in order to find the magnetic bearings with which to compare his compass bearings.

22. If a good compass is used at the shore station, and its position may be regarded as free from any peculiar local disturbance, the bearings observed with it may be assumed as the true magnetic bearings of the ship's compass; and the differences of the opposites of these from the compass bearings observed on board, may be taken as the deviations, and tabulated or plotted.

The deviation is east, if the bearing by the shore compass is to the right of the corresponding bearing by the ship's compass; west, if the bearing by the shore compass is to the left of the corresponding bearing by the ship's compass.

This method is generally preferred, especially for iron ships, where the local disturbance is large. It assumes that the shore compass gives the magnetic bearing more truly than the mean resultant of the observations made on board.

23. At sea the observations may be made on the sun, and to better advantage when it is near the horizon. Its true azimuth at each instant of observing its bearing by the compass must be found either—

First, By means of simultaneous altitudes; or,

Second, By noting the local apparent times; or,

Third, By altitudes at equal intervals of 10 m., 20 m., or 30 m., and the computed azimuths interpolated for the time of the compass observation.

The differences of the true azimuths from the compass bearings will be the declination combined with the deviation. The mean of a series of observations made at equal intervals round the compass will be the declination of the compass used; the differences from that mean, the deviations for the several directions of the ship's head. A graphical process may be used similar to that described in Art. 20, and advantageously when the series of observations is not symmetrical, or any of them have been omitted.

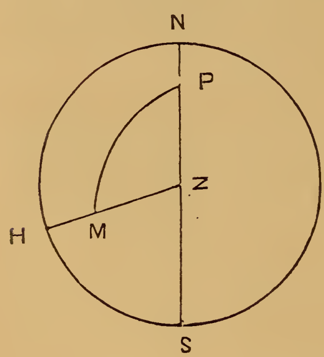
24. To allow time for the needle to settle, and for several bear-

ings to be taken, it is desirable to keep the ship's head steady for a few minutes at each of the points selected.

Unless the deviations are great, observations on each of the sixteen principal points are sufficient. Observations carefully made on eight points may give a result sufficiently accurate for the ordinary purposes of navigation. Very careful observations on the four following points (by compass), N.E., S.E., S.W., and N.W., have sometimes been used in default of others.

25. Prob. 1.—To find from an observed altitude the true azimuth of a heavenly body at any place, the Greenwich time of observation being known.

FIG. 29.



We have as in the figure (29) the following given :

$PM = p = 90° - d$
$PZ = 90° - L$
$ZM = z = 90° - h$
to find the angle PZM.

From Spher. Trig. 164, 165, 166, we have

$$\sin \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}}$$

$$\cos \tfrac{1}{2} A = \sqrt{\frac{\sin s \sin (s-a)}{\sin b \sin c}}$$

$$\tan \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}}$$

Using the formula for the sine, and these values of the sides, viz. :

$Co\ L$, $90° - h$, and $90° - d$

we will have

$$\sin \tfrac{1}{2} Z = \sqrt{\frac{\cos \frac{1}{2} (Co\ L + h + d) \sin \frac{1}{2} (Co\ L + h - d)}{\cos L \cos h}}$$

or, if we put

$$S' = \tfrac{1}{2}\,(Co\ L + h + d).$$

$$\sin \tfrac{1}{2}\, Z = \sqrt{\frac{\cos S' \sin\,(S' - d)}{\cos L \cos h}} \qquad (a.)$$

Using the formula for the cosine and the following values for the sides,

$$90^\circ - L,\ 90^\circ - h \text{ and } p$$

we have

$$\cos \tfrac{1}{2}\, Z = \sqrt{\frac{\cos \tfrac{1}{2}\,(L + h + p)\, \cos \tfrac{1}{2}\,(L + h - p)}{\cos L \cos h}}$$

or if we put

$$S'' = \tfrac{1}{2}\,(L + h + p)$$

$$\cos \tfrac{1}{2}\, Z = \sqrt{\frac{\cos S'' \cos\,(S'' - p)}{\cos L \cos h}} \qquad (b.)$$

Using the formula for the tangent, with the following values of the sides, viz.:

$$Co\ L,\ p \text{ and } z,$$

and putting

$$S'' = \tfrac{1}{2}\,(Co\ L + p + z)$$

we have

$$\tan \tfrac{1}{2}\, Z = \sqrt{\frac{\sin S''' - Co\ L)\, \sin\,(S''' - z)}{\sin S''' \sin\,(S''' - p)}}$$

When

Z is less than 90° use (a).

When

Z is greater than 90° use (b).

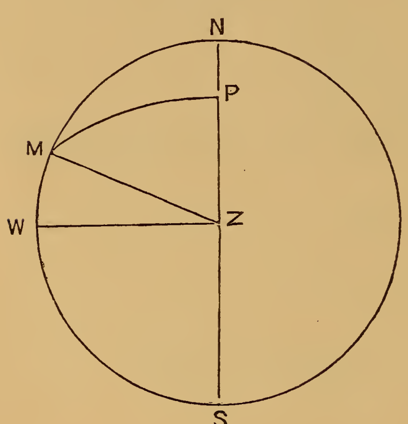
If greater accuracy is desired than is generally necessary at sea, use (c).

The formula for the $\cos \frac{1}{2} Z$ is generally used in case of the sun in connection with A. M. and P. M. time sights. The data required is the same as that for determining the hour angle. Z is the true bearing or azimuth of the body, reckoned from the north point of the horizon in north latitude, and from the south point in south latitude. If reckoned as positive toward the east, it must be negative toward the west.

It is generally best to use the supplement of Z when it is greater than 90°, as the readings of azimuth compasses are from 0° to 90°. If, when the altitudes are observed, the bearings of the heavenly body be taken by an azimuth compass, by comparing the magnetic and true bearings we may obtain the *variation and deviation of the compass* combined. It is marked E when the true bearing is to the *right* of the magnetic bearing, otherwise W.

26. Problem 2.—To find the amplitude and azimuth of a heavenly body when in the horizon, the Greenwich time being given.

FIG. 30.



In Fig. 30, the body M being in the true horizon, $W\,M$ is its amplitude, NM its azimuth.

In the triangle $P\,N\,M$, right angle at N, we have.

$$\cos P\,M = \cos M\,N \cos P\,N$$

$$\cos p = \sin d = \cos Z \cos L.$$

If $a =$ amplitude $= 90° - Z$

$$\cos Z = \sin a = \sin d \sec L$$

27. Problem 3.—To find the altitude and azimuth of a heavenly body at a given place and time.

In Fig. 31 we have given

$P\,Z = 90° - L$

$Z\,P\,M = t$, the hour angle of body M.

$P\,M = 90° - d$, to find

$Z\,M = 90° - h$, and

$P\,Z\,M = Z$ the azimuth.

$$\cos t = \cot \phi'' \tan d$$

$$\tan \phi'' = \tan d \sec t \qquad (a.)$$

$$\phi' = \phi'' - L$$

$$\sin d : \sin h = \sin \phi'' : \cos (\phi - L$$

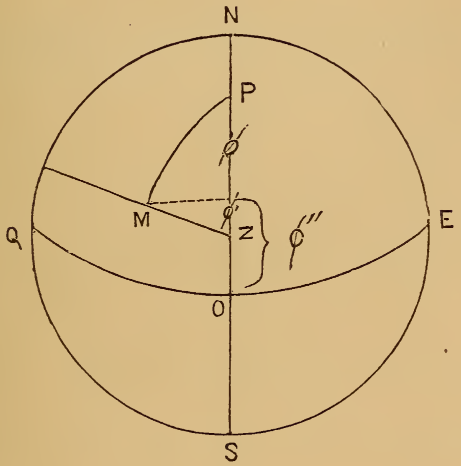
$$\sin h = \frac{\cos (\phi'' - L) \sin d}{\sin \phi''} \qquad (b.)$$

$$\cos \phi'' : \sin (\phi'' - L) = \cot t : \cot Z$$

$$\cot Z = \frac{\sin (\phi'' - L) \cot t}{\cos \phi''} \qquad (c.)$$

ϕ'' is marked N. or S. like the declination, and is the same quadrant as t (numerically).

FIG. 31.



In (a) if $t = 6$ h. $\phi'' = 90°$ and (c) assumes the indeterminate form; from (a) we have, however

$$\cot t = \frac{\tan d}{\tan \phi'' \sin t}$$

which substituted in (c) gives

$$\cot Z = \frac{\sin (\phi'' - L) \tan d}{\sin \phi'' \sin t}$$

which may be used when $t = 6$ h. nearly.

Z is the *true* bearing of the body reckoned from the elevated pole. The negative value need not be used, however, by restricting Z numerically to 180°, and marking it E or W like t.

Prob. 4.—To find the altitude when azimuth is not required.

We have

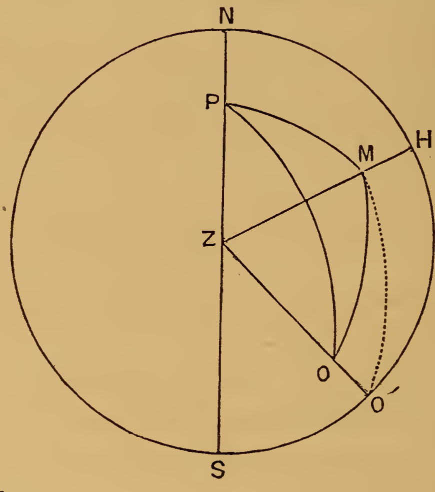
$$\sin h = \sin L \sin d + \cos L \cos d \cos t$$
$$\cos t = 1 - \text{versin}\, t.$$

Which substituted gives

$$\sin h = \sin L \sin d + \cos L \cos d - \cos L \cos d \text{ versin}\, t.$$
$$\sin h = \cos (L - d) - \cos L \cos d \text{ versin}\, t.$$

Prob. 5.—To find the azimuth or true bearing of a terrestrial object.

FIG. 32.



In Fig. 32, let

Z be the zenith or place of the observer ;
O the terrestrial object ;
M the apparent place of some heavenly body ;
Z its azimuth ;
z the angle *M Z O*,

or azimuth angle between the heavenly body and the object. This angle may be obtained by direct measurement with a theodolite, plane table, or graduated top of azimuth compass.

Z is found as in Prob. 1, and we would have for the azimuth of the terrestrial object O

$$N\,Z\,O = Z + z.$$

Another method of determining the angle z, is by measuring with a sextant or arc $M\,O$, noting the time and measuring simultaneously the altitude of M. Then measure the altitude of the terrestrial object O.

Let $H' = 90° - Z\,M$, the *apparent* altitude of M,
$h' = 90° - Z\,O$, the apparent altitude of O,
D, the distance $M\,O$, corrected for index error of sextant, and semidiameter of heavenly body.

We have then in the triangle $M\,Z\,O$, the three sides given to find $z = M\,Z\,O$.

Using formula from Trigonometry for sine we obtain

$$\sin \tfrac{1}{2} Z = \sqrt{\frac{\sin \frac{1}{2}(D + H' - h') \sin \frac{1}{2}(D - H' + h')}{\cos H' \cos h}}$$

for cosine

$$\cos \tfrac{1}{2} Z = \sqrt{\frac{\cos \frac{1}{2}(H' + h' + D) \cos \frac{1}{2}(H' + h - D)}{\cos H' \cos h'}}$$

If O is in the true horizon, or its measured altitude equals the dip, the right triangle $M\,H\,O'$ gives

$$\cos z = \cos H\,O' = \cos D \sec H'$$

z, thus determined after sextant measurement, may be applied as before to the computed azimuth of M, to obtain the azimuth of the terrestrial object.

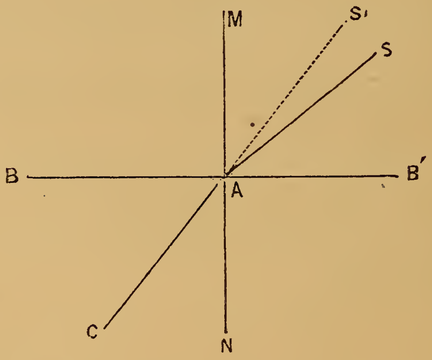
CHAPTER VIII.

REFRACTION.—DIP.—PARALLAX, AND SEMIDIAMETER.

1. When a ray of light passes *obliquely* from one medium to another of different density, it is bent or *refracted* from a rectilinear course. The ray before it enters the second medium is called the incident ray, afterwards the refracted ray. The difference between the directions of these two rays is the *refraction.*

The angle which the incident ray makes with a normal to the surface of the refracting medium, when the incident ray meets it, is called the *angle of incidence.* The angle which the refracted ray makes with the normal is the *angle of refraction.* The difference between these two angles is therefore the refraction.

Fig. 33.



In the figure (33), if $S\ A$ is an incident ray upon the surface $B\ B'$ of a refracting medium, $A\ C$ the refracted ray, and $M\ N$ A normal to the surface at A, $S\ A\ M$ is the angle of incidence, $C\ A\ N$ or $S'\ A\ M$ is the angle of refraction, and $S\ A\ S'$ the refraction. An observer situated anywhere along the line $A\ C$ will receive the ray as if it had come directly to his eye without refraction from S'. $S' A C$ is called the *apparent* direction of the ray.

2. It is shown in works upon optics, that refraction take place according to the following general laws :

1*st.* When a ray of light falls upon a surface of any form, which separates two media of different densities, the incident ray, refracted ray, and normal to that surface at the point of incidence, are in one plane.

2*d.* When a ray passes from a rarer to a denser medium, it is refracted *towards* the normal; and when a ray passes from a denser to a rarer medium it is refracted *from* the normal.

3*d.* When the densities of the two media are constant, there is a constant ratio between the sine of the angle of incidence and the sine of the angle of refraction. If a ray passes from a vacuum into a given medium, the number expressing this constant ratio is called the *index of refraction* for that medium. This index is always an improper fraction, being equal to the sine of the angle of incidence divided by the sine of the angle of refraction.

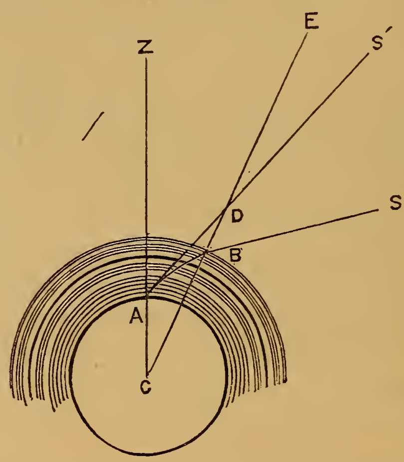
4*th.* When a ray passes from one medium into another, the sines of the angles of incidence and refraction are reciprocally proportional to the indices of refraction of the two media.

3. **Astronomical Refraction.**—The rays of light from a heavenly body in coming to the observer must pass through our atmosphere. If the space between the star and the upper limit of the atmosphere be regarded as a vacuum, or as filled with a medium which exerts no sensible effect upon the direction of a ray of light, the path of the ray until it reaches the atmosphere, will be a straight line; but upon entering the atmosphere will be refracted *towards* the normal to the surface of the atmosphere at the point of incidence. The atmosphere not being of uniform density, the ray is *continually* passing from a rarer to a denser medium, so that its path becomes a curve concave towards the earth.

The *apparent* direction of the ray will be that of a tangent to the curve at the point where it reaches the eye. The difference in direction of this tangent and the ray before it reaches the atmosphere is called the astronomical refraction.

The ray (Fig. 34) from the star S entering the earth's atmosphere at B is bent into the curve $A\ B$.

Fig. 34.



The observer at A sees it in the direction of the tangent $A\ S'$. From the first law given, the vertical plane of the observer which contains the tangent $A\ S'$ must also contain the normal $E\ C$ and the incident ray $B\ S$. Hence refraction increases the altitude of a heavenly body without changing its azimuth.

The angle $Z\ A\ S$ is the *apparent zenith distance* of the heavenly body. The angle $E\ B\ S$ is the angle of refraction, and $Z\ A\ S$, the apparent zenith distance, is the angle of refraction. If we represent the refraction by r, we have

$$r = E\ B\ S - E\ D\ S'$$

and from the third law

$$\frac{\sin E\ B\ S}{\sin Z\ A\ S'} = m,$$

a constant ratio for a given condition of the atmosphere and a given position of A.

4. **To find the refraction r.**

In the figure, let

$z = Z\,A\,S'$, the apparent zenith distance,
$r = E\,B\,S - E\,D\,S'$, the refraction,
$u = Z\,C\,E$,

Then

$$E\,D\,S' = A\,D\,C = Z\,A\,S' - Z\,C\,E = z - u$$
$$E\,B\,S = Z - u + r$$
$$\frac{\sin E\,B\,S}{\sin Z\,A\,S'} = \frac{\sin\,(z - u + r)}{\sin Z} = m$$
$$\sin\,[z - u - r)] = m \sin z.$$
$$\frac{\sin\,[z - (u - r)] + \sin z}{\sin\,[z - (u - r)] - \sin z} = \frac{m + 1}{m - 1}$$

which by (109) Plane Trigonometry becomes

$$\frac{\tan \tfrac{1}{2}\,[z - (u - r) + z]}{\tan \tfrac{1}{2}\,[z - (u - r) - z]} = \frac{m + 1}{m - 1}$$

which reduces to

$$\frac{\tan\,[z - \tfrac{1}{2}\,(u - r)]}{\tan \tfrac{1}{2} - (u - r)} = \frac{m + 1}{m - 1}$$

hence

$$\tan \tfrac{1}{2}\,(u - r) = \frac{1 - m}{1 + m} \tan\,[z - \tfrac{1}{2}\,(u - r)] \qquad (a.)$$

In this u and r are both unknown, but are both small angles, being 0 when the zenith distance is 0, and increasing with the zenith distance. Assuming that they vary proportionately, and that

$$\frac{u}{r} = q$$

and substituting in (a) we have

$$\tan \tfrac{1}{2}\,(q - 1)\,r = \frac{1 - m}{1 + m} \tan \tfrac{1}{2}\,[z - \tfrac{1}{2}\,(q - 1)\,r]$$

as $\frac{1}{2}\,(q - 1)\,r$ is very small we may put

$$\tan \tfrac{1}{2}\,(q - 1)\,r = \tfrac{1}{2}\,(q - 1)\,r \sin 1''$$

and have

$$\tfrac{1}{2}(q-1)\,r \sin 1'' = \frac{1-m}{1+m} \tan\left[z - \tfrac{1}{2}(q-1)\,r\right]$$

whence

$$r = \frac{2}{q-1 \sin 1''} \cdot \frac{1-m}{1+m} \tan\left[z - \tfrac{1}{2}(q-1)\,r\right]$$

Putting

$$n = \frac{2}{(q-1)\sin 1''} \cdot \frac{1-m}{1+m}$$

and

$$p = \tfrac{1}{2}(q-1)$$

we have

$$r = u \tan (z - p\,r)$$

which is known as Bradley's Formula.

If at two given zenith distances z' and z'' the refractions r' and r'' are formed by observations in a mean state of the atmosphere, then we have the two equations

$$r' = n \tan (z' - p\,r'),$$
$$r'' = n \tan (z'' - p\,r'');$$

and the two unknown quantities n and p may be found.

By comparing observations in this way at various zenith distances, the values of n and p are found to be very nearly the same; so that the assumption made is found to be nearly correct.

The values of n and p used in the computation of Table XII. (Bowd.) are

$$n = 57''.036 \text{ and } p = 3$$

These values correspond to the height of

the barometer, $b = 29.6$ inches,
the thermometer, $t = 50°$ Fahr.

5. Refraction in different conditions of the atmosphere is nearly proportional to the density of the air; and this density, the temperature being constant in proportional to its elasticity; that is, to the height of the barometer. Then, if

$\varDelta$ is the noted height of the barometer,
r, the refraction of Tab. XII.
$\varDelta\, r$, the barometer correction

$$\frac{r + \varDelta r}{r} = \frac{b}{29.6}$$

$$r + \varDelta r = \frac{b}{29.6} r$$

$$\varDelta r = \frac{b}{29.6} r - r$$

$$\varDelta r = \left(\frac{b}{29.6} - 1\right) r$$

$$\varDelta r = \frac{p - 29.6}{29.6} r$$

The correction for the barometer in Table XXXVI. (Bowd.) is computed from the formulæ.

The elastic force being constant, the density increases by $\frac{1}{400}$ part for each degree of depression of the thermometer (Fahr.).

Hence, if

$\varDelta' r =$ the correction for the thermometer,
$t =$ the noted temperature

$$\varDelta' r = \frac{50° - t}{400} (r + \varDelta' r)$$

$$400'' \varDelta' r = (50° - t(\ (r + \varDelta' \text{r})$$

$$= 50° r + 50° \varDelta' r - t r - \varDelta' r t$$

$$350° \varDelta r' + \varDelta' r t = 50° r - t r$$

$$\varDelta' r = \frac{50° - t}{350° + t} r,$$

by which the correction for thermometer, Tab. XXXVI. (Bowd.), is computed.

7. To find the radius of curvature of the path of a ray in the earth's atmosphere.

By the radius of curvature, is meant the radius of a circle which most nearly coincides with the curve.

If in Fig. 35 we consider the curvature to be uniform from B to A, the problem is reduced to finding the radius of this arc.

Let C' be the centre of the arc $A B$,
$R' = C' A$, the radius of curvature,
$R = C A$, the radius of the earth.

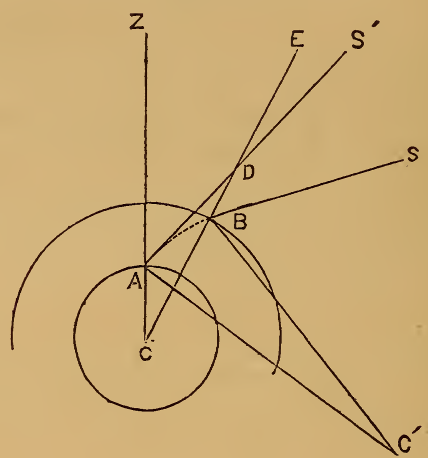
$S\,B'$ and $S\,A'$ are tangents to the curve at the points B and A respectively. The angle between the radii $C'\,A$ and $C'\,B$ is equal to the angle made by these tangents with each other, which is the refraction r. As $A\,B$ is a very small arc, we may put

$$A\,B = R' \sin r$$

and nearly

$$A\,D = A\,B = R' \sin r$$

FIG. 35.



In the triangle $A\,D\,C$

$$\frac{R' \sin r}{R} = \frac{\sin u}{\sin (z - u)}$$

whence

$$R' = \frac{R}{\sin (u - r)} \cdot \frac{\sin u}{\sin r}$$

and as u and r are small

$$R' = \frac{R}{\sin z} \cdot \frac{u}{r}.$$

But by preceding work

$$\frac{u}{r} = q \text{ and } p = \tfrac{1}{2}(q - 1) = 3$$

whence

$$q = 7, \qquad u = 7\,r$$

so that

$$R' = \frac{7\,R}{\sin z}\;.$$

When

$$z = 0, \text{ or the star is at the zenith,}$$
$$R' = \infty$$

When

$$z = 90°, \text{ or the star is in the horizon,}$$
$$R' = 7\,R$$

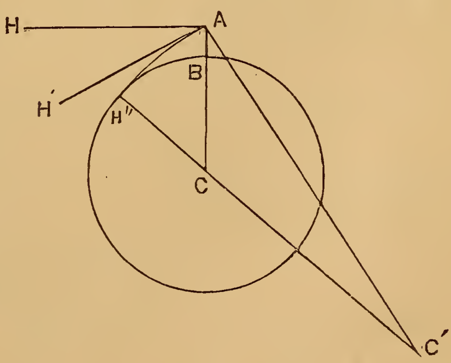
This is for a mean condition of the atmosphere for which the values of p and q were obtained. The curve is greatly varied for extraordinary states of the atmosphere.

We have seen that infraction increases the apparent altitude of a heavenly body. As a correction, therefore, to an observed altitude, to obtain true altitudes, it is always subtractive.

DIP.

8. A plane, tangent to the earth's surface, is called the true horizon. If an observer be elevated above the plane, the visual ray will be tangent at some other point on the earth's surface. If it were not for the effect of refraction, the angle between the visual ray and the true horizon would be a correction to be applied to an observed altitude to obtain true altitudes. The effect of refraction is to determine this angle.

FIG. 36.



In Fig. 36, the most distant point visible from A is H'' where the visual ray $A\ H''$ is tangent to the earth's surface. The apparent direction of H'' is $A\ H'$. $H\ A\ H''$ is called dip of the horizon. It increases in apparent altitude, and as a correction is subtractive.

9. To find the dip of the horizon.

Let C be the centre of the earth

C' the centre of the arc $H''\ A$

$H''\ C\ C'$ are in the same straight line, since the arcs

$H''\ A$ and $H''\ B$ are tangent to each other at H''

$C\ A$ and $C'\ A$ are perpendicular respectively to $A\ H$ and $A\ H'$;

hence

$$H\ A\ H' = C\ A\ C' = \varDelta\ H, \text{ the dip.}$$

If $h =$ the height $B\ A$ above the sea level,

$C\,A = R + h$

$C'\ A = 7\ R$, the radius of curvature of the arc $H''\ A$

$C\ C' = 6\ R$.

In the triangle $C\ A\ C'$, by Plane Trigonometry, we have

$$\sin \tfrac{1}{2}\ \varDelta\ H = \sqrt{\frac{(6\ R - \frac{1}{2}\ h)\ (\frac{1}{2}\ h)}{7\ R\,(R + h)}}$$

h is so small that it may be omitted when additive to R, and we have

$$\sin \tfrac{1}{2}\ \varDelta\,H = \sqrt{\frac{3\ h}{7\ R}}$$

As $\frac{1}{2}\ \Delta\ H$ is a small angle

$$\tfrac{1}{2}\ \varDelta\ H \sin 1'' = \sqrt{\frac{3\ h}{7\ R}}$$

$$\varDelta\ H = \frac{2}{\sin\ 1''} \sqrt{\frac{3\ h}{7\ R}} = \frac{2}{\sin\ 1'} \sqrt{\frac{3}{7\ R}} \sqrt{h}$$

$\dfrac{2}{\sin\ 1''} \sqrt{\dfrac{3}{7\ R}}$ is a constant and may be computed. Its value will depend upon the value of R used. Bowditch uses the value in Vince's Astronomy. The logarithm of the constant used by him is 1.7712711.

$$\log \Delta\ H = 1.7712711 + \tfrac{1}{2} \log h$$

h is expressed in feet, and $\Delta\ H$ found in seconds.

10. To find the distance of an object of known height just visible in the horizon.

In figure of previous Art.

$h = B\ A$, the height of A

$d = H''\ A$, the distance of A.

As this arc is small, we shall have

$$d = H''\ C'\ A \sin 1'' \times C\ A = 7\ R \times H''\ C'\ A \sin 1'' \qquad (a.)$$

In the triangle $C\ C'\ A$, we have

$$\sin \tfrac{1}{2}\ H'\ C'\ A = \sqrt{\frac{\tfrac{1}{2}\ h\ (R + \tfrac{1}{2}\ h)}{42\ R^2}},$$

or nearly

$$\tfrac{1}{2}\ H'\ C'\ A \sin 1'' = \sqrt{\frac{h}{84\ R}}$$

$$H''\ C'\ A \sin 1'' = \sqrt{\frac{h}{21\ R}}$$

which substituted in (a) gives

$$d = 7\ R\ \sqrt{\frac{h}{21\ R}} = \sqrt{7/3\ R\ h}$$

If d, h and R are expressed in feet, in geographical miles

$$d = \frac{1}{6087}\sqrt{7/3\ R\ h}$$

Table X., Bowd., is computed for d in statute miles. It would be more useful to the Navigator if it were in geographical miles.

PARALLAX.

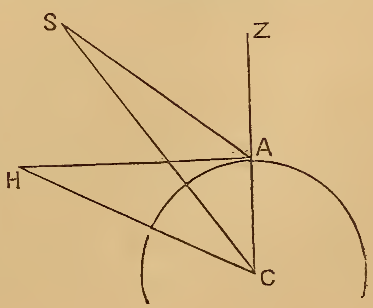
11. Change in direction due to change of position is called *Parallax.* In astronomical observations, the observer is on the surface of the earth. It is convenient to reduce them to the earth's centre. The change in direction of a heavenly body, as viewed from the earth's surface and from its centre, is called *geocentric parallax.* Geocentric parallax may be defined as the angle at the body subtended by that radius of the earth which passes through the observer's position.

In Fig. 37, the geocentric parallax of the body S will be

$$S = Z\ A\ S - Z\ C\ S$$

This is regarding the earth as a sphere, which is sufficiently accurate for all nautical problems except the complete reduction of lunar distances, when the spheroidal form of the earth must be taken into consideration.

FIG. 37.



12. To find the parallax of a body in the horizon *H*.

Let π = the parallax, called in this case the *horizontal parallax*,
d, the distance of the body from the centre of the earth,

then

$$\sin \pi = \frac{R}{d}$$

13. To find parallax of a heavenly body for a given altitude.

In the triangle $C\ S\ A$, letting p = the parallax, we have

$$\sin p = \frac{R \sin z}{d}$$

Substituting in this the value of the horizontal parallax, gives

$$\sin p = \sin \pi \sin z$$

or nearly, as π and p are small angles,

$$p = \pi \sin z$$
$$p = \pi \cos h$$

The horizontal parallax π is given in the Nautical Almanac for the sun, moon, and planets. From the figure it is evidently the semidiameter of the earth as viewed from the body. As the equatorial semidiameter of the earth is larger than any other, so will be the *equatorial horizontal parallax*. This the Nautical Almanac gives for the moon. For refined observations this will have to be reduced for the latitude of the observer.

Tables X., A., and XIV. are computed by the above formulæ.

Table XIX., Bowd., contains a quantity to be subtracted from 59′ 42″, the remainder being the combined corrections of parallax and refraction for the moon's altitude.

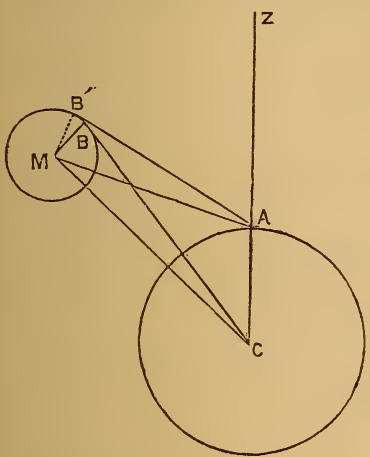
APPARENT SEMIDIAMETERS.

14. The apparent semidiameter of a body is the angle subtended by its radius at the place of the observer. Observations of the sun and moon with sextant are made by bringing either the upper or lower limb in contact with the sea horizon, or (in using the artificial horizon) by bringing two opposite limbs of direct and reflected limbs together. The altitude of the centre

of the body being required, the angular semidiameter of the heavenly body must be applied plus or minus, according to the limb observed.

15. To find the apparent semidiameter of a heavenly body.

FIG. 38.



In Fig. 38,

Let M be the body,

$d = C\,M$, its distance from earth's centre,

$d' = A\,M$, its distance from A

$S = M\,C\,B$, its apparent semidiameter as viewed from C

$S' = M\,A\,B'$, its apparent semidiameter as viewed from A

$R = C\,A$, the earth's radius

$r = M\,B = M\,B'$, the linear radius of the body.

For finding S, the right triangle $C\,B\,M$ gives

$$\sin S = \frac{r}{d} \tag{a.}$$

Were the body M in the horizon of A, its distance from A and C would be sensibly the same, so the angle S is called the *horizontal* semidiameter.

From Art. 12, we have for the horizontal parallax

$$\sin \pi = \frac{R}{d} \text{ or } d = \frac{R}{\sin \pi}$$

which substituted in (a) gives

$$\sin S = \frac{r}{R} \sin \pi$$

or

$$S = \frac{r}{R}\,\pi$$

$\frac{r}{R}$ is constant for any particular body, and representing it by m, we have

$$\log S = \log m + \log \pi$$

The Nautical Almanac gives the semidiameters of the sun, moon and planets.

16. To find S', the apparent semidiameter as seen from A, the right triangle $A\ B'\ M$ gives

$$\sin S' = \frac{r}{d'} \qquad (b.)$$

In the triangle $C\ M\ A$

$$\frac{\sin M\ A\ C}{\sin M\ C\ A} = \frac{d}{d}$$

If

$h = 90° - Z\ A\ M$, the apparent,
$h' = 90° - Z\ C\ M$, the true altitude of M.

$$\frac{\cos h}{\cos h'} = \frac{d}{d'}$$

$$d' = d\,\frac{\cos h'}{\cos h}$$

which substituted in (b) gives

$$\sin S' = \frac{r \cos h}{d \cos h'}$$

substituting for $\frac{r}{d}$ its value from (a) we have

$$\sin S' = \sin S\,\frac{\cos h}{\cos h'}$$

$$S' = S\,\frac{\cos h}{\cos h'}$$

gives an approximate value for S', when S and h are known.

As $h < h'$, $\cos h > \cos h'$ and consequently $S' > S$, or the semidiameter increases with the altitude of the body. This excess is called the augmentation, and is only sensible in the case of the moon.

17. **To find the augmentation of the moon's semidiameter.**

$$S' - S = \Delta\ S = S\,\frac{\cos h - \cos h'}{\cos h'}$$

which by Plane Trigonometry (108) becomes

$$\Delta S = S \frac{2 \sin \frac{1}{2} (h' + h) \sin \frac{1}{2} (h' - h)}{\cos h'}$$

$h' - h = p$, the parallax, and being small

$$2 \sin \tfrac{1}{2} (h' - h) = 2 \sin \tfrac{1}{2} p = p \sin 1'' = \pi \cos h \sin 1''$$

and as $\varDelta S$ is small, we may take $\frac{1}{2} (h + h') = h$, and $\cos h$ for $\cos h'$; and then

$$\varDelta S = S \pi \sin 1'' \sin h$$

and as

$$S = \frac{r}{R} \pi$$

$$\varDelta S = \frac{r}{R} \pi^2 \sin 1'' \sin h.$$

For the moon

$$\frac{r}{R} = 0.2729 : \text{then}$$

$$\varDelta S = .000001323 \, \pi^2 \sin h.$$

Using the mean value of $\pi = 57' \, 20''$.

$$\varDelta S = 15''.65 \sin h.$$

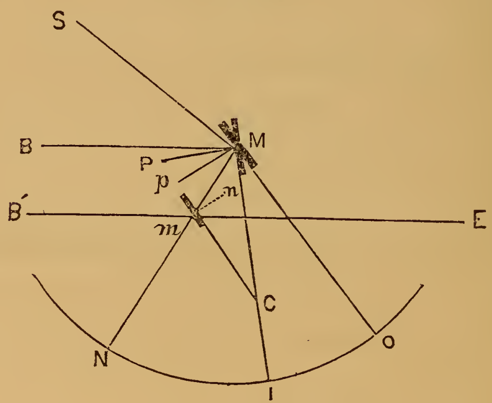
Tab. XV. (Bowd.) is computed from a formula nearly like this.

CHAPTER IX.

SEXTANT.—ARTIFICIAL HORIZON.

1. The optical principle of the construction of the sextant is the following: "If a ray of light suffers two successive reflections in the same plane by two plane mirrors, the angle between the first and last directions of the ray is equal to twice the angle of the two mirrors."

FIG. 39.



In Fig. 39, let M and m be the two mirrors. The direct and reflected rays are always found in the same plane—called the plane of reflection. In order that the last direction of the ray after suffering *two* reflections shall be in the same plane as the first direction, the plane of reflection must be perpendicular to both mirrors. In the diagram the plane of the paper is the plane of reflection. The shaded lines M and m are the intersections of this plane with the mirrors. Let $S\,M$ be the direct ray falling upon the mirror M (lying in the direction $M\,I$). Let $M\,m$ be the direction of the ray after the first reflection, and $m\,E$ its direction after the second reflection. Draw $M\,B$ parallel to $m\ E$, $M\ P$ perpendicular to $M\ C$, and $M\ p$ perpendicular to the mirror m. The angle $S\ M\ B$ is the angle between the rays $S\ M$ and $m\ E$. The angle $P\ Mp$, being obviously equal to $M\ C\ m$, is the angle between the mirrors. We have, then, to prove $S\ MB = 2\ P\ M\ p$.

If m, draw the perpendicular $m\ n$, $M\ m\ n = p\ M\ m$, is the angle of incidence of the ray $M\ m$ on the mirror m; $n\ m\ E = p\ M\ B$ is the angle of reflection of the same ray. The angle of incidence and the angle of reflection being equal, we have

$$p\ M\ m = p\ M\ B = P\ M\ p + P\ M\ B$$

On the same principle we have

$$P\ M\ m = P\ M\ S = S\ M\ B + P\ M\ B$$

Taking the difference of these two equations we have

$$P\ M\ p = S\ M\ B - P\ M\ p$$

hence

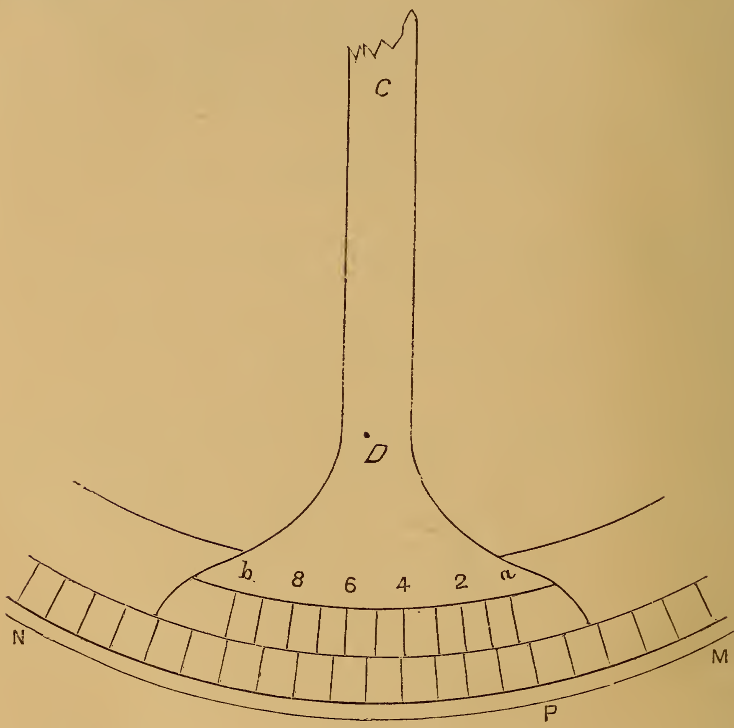
$$S\ M\ B = 2\ P\ M\ p = 2\ M\ C\ m.$$

2. This principle is applied in the sextant as follows: The mirror M is attached to a bar $M\ I$, called the index bar, which revolves upon a pivot at M in the centre of a graduated arc $O\ N$. The mirror M is firmly fixed at right angles to the plane of this arc. The mirror M is called the *index glass*; the mirror m the *horizon glass*. Place the index bar in the position $M\ O$ so that the two glasses are parallel. In this position an incident ray from an object B will be reflected first to $n\ o$ and then in the direction $m\ E$. The first and last directions of the ray will be parallel. If, then, the object is so distant that two rays from it, $B\ M$ and $B'\ m$, falling upon the two mirrors are sensibly parallel, the the observer at E will receive the direct and reflected ray at the same time, or will see two images of the same object in *coincidence*. Commence the graduation of the limb at O, marking it zero. Move the index bar to the position $M\ I$, so that a ray from the object S is reflected in the direction $m\ E$; the observer E sees the object B and S in coincidence, and the angle $S\ M\ B$

between the two objects is equal to twice the angle through which the index glass has been moved.

As the centre of rotation is at *M*, this angle will be twice the angle *O M I*.

If, now, the arc *O N* be graduated, and the marking of the graduation doubled, we can read at once the angle *S M B*. The angles are read to a nicety by means of a *vernier*, on the index bar at *I*.



THE VERNIER.

3. Let *M N*, Fig. 40, be a portion of the limit of a circle, *C D* the arm which revolves with the index glass about the centre of the circle. At the end of this arm, construct *a b* graduated into a number of divisions which occupy the space of *n* 1 of the limb. The first line *a* is the zero of the *vernier*, and the reading is to be determined from the position of this zero on the limb *M N*.

If we put

$x =$ the value of a division of the limb.

$y =$ the value of a division of the vernier, we will have

$$(n-1)\, x = n\, y$$

hence

$$y = \frac{n-1}{n}\, x$$

and

$$x - y = \frac{1}{n}\, x$$

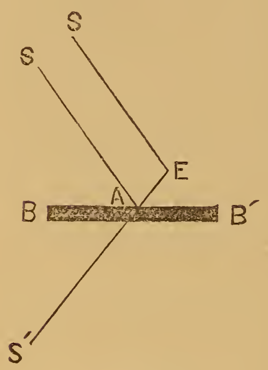
The difference $x - y$ is called the *least count* of the vernier, which is, therefore, $\frac{1}{n}$th of a division of the limb. If the zero of the vernier falls between the two graduations P and $P + 1$, the whole reading is P plus the fraction from P to a. To measure this fraction, m, observe that if the mth division of the vernier is in coincidence with a division of the limb, the fraction is $m \times (x - y)$ or $\frac{m}{n}\, x$. In the figure the vernier is divided into *ten* equal parts, equal to *nine* divisions of the limb, and if the 4th division is in coincidence, the whole reading is $P + \frac{4}{10}\, x$; and if $x = 10'$, then the whole reading is $P + \frac{4}{10} . 10 = P + 4'$. Suppose that P is the division of the limb marked 35° 40′, then the reading is 35° 44′. The least count in this case is 1′. The fraction is obtained in practice by the numbers placed above (or below) the divisions on the vernier.

Sextants generally read to 10″; in other words the *least count* is 10″. From the above it will be seen that for this 60 divisions of the vernier equal 59 divisions of the limb. Verniers are sometimes constructed (seldom for sextants) with the divisions on the vernier greater than those upon the limb. The only difference will be that the reading of the vernier will be in a direction opposite to that of the reading of the limb.

For the adjustments of the sextant see Chauvenet's Astronomy, pp. 95 to 99, inclusive, or Bowd., pp. 133-136. *Circles of reflection* and *octants* are similar in construction to the sextant.

4. The **artificial horizon** is a small basin partially filled with mercury, over which is placed a roof consisting of two plates of glass fitted in a frame at right angles to each other. The roof is to protect the surface of the mercury from wind and dust. The best form have a wooden basin fitting inside of a metallic one. A small funnel screws into a hole at one end of the wooden basin; a channel underneath conveys the mercury to the centre of the basin. The funnel acts as a strainer, retaining a greater portion of the oxide. If the mercury be amalgamated with tin, all impurities will float upon the surface, and may be removed by passing lightly over the surface the edge of a piece of paper.

Fig. 41.



If, in Fig. 41, $B\ B'$ be the horizontal surface of the mercury, $S\ A$ a ray of light from a heavenly body incident upon the surface at A, it will appear to an observer at E in the direction $S'\ E$. The angular depression $B\ A\ S'$ below the horizontal plane is equal to $S\ A\ B$, the altitude above this plane. If, then, $S\ E$ is a direct ray from the heavenly body parallel to $S\ A$, and the observer at E with a sextant makes the direct image S and the reflected image S' coincide, the reading of the sextant will be $S\ E\ S' = S\ A\ S' = 2\ S\ A\ B$. The surface of the mercury being in the plane of the true horizon, the altitude obtained has only to be corrected for parallax and refraction, and in case the limit of a body has been observed, for semidiameter. The index correction of the sextant, as is obvious, must be applied to the reading of the sextant. Parallax and refraction to the altitude of the body, and semidiameter to the altitude or *diameter* to the reading of the sextant. The glasses in the roof should be made of plate glass with parallel faces. To eliminate any error that may arise from a prismatic form of the glasses, observe one half of a set of altitudes with one end of the roof towards the observer, and one half with the other end towards the observer. In the case of *equal altitudes*, keep the same end towards the observer.

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